Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 234 JULY, 1942 No. 1

LOOKING WESTWARD.*

BY

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Long before the Declaration of Independence, two entirely different and opposite views of the future of America manifested themselves in the British colonies. They corresponded in fact to two different philosophies of life: one reflecting what might be called an old world or European point of view; the other one already embodying some of the most essential features of what was to become the doctrine of Americanism.

It has been often observed that men modify very slowly their intellectual habits even in new surroundings. Despite new conditions and new ways of life, for a long time, many colonists failed to realize the limitless potentialities of the country in which they had established "plantations." Their thoughts continued to run in accustomed channels, even when they began to think of the new settlements as a sort of unit having an entity and rights of its own. One of the common sense conclusions, which had been reached before Montesquieu had given to it a final and almost axiomatic form, was that great danger lay in territorial expansion and in mere size. Between the ideal of a perfectly ordained society,

*Delivered in the Hall of The Franklin Institute, January 19, 1940. This was one of the papers in the "Meet Dr. Franklin" Conference.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

which prevailed chiefly in New England, and the vast expanse of land extending beyond the horizon, existed a sort of antinomy. For various reasons, they had left the mother country, but they turned their eyes towards Europe; one of their chief preoccupations was to implant in the howling wilderness a form of life as closely resembling English life as they could. For generations they had watched the wars waged on the continent and had participated in them. The lessons of history and experience were clear: a small population occupying a large territory was at the mercy of powerful and envious neighbors. They had learned that frontiers have to be protected and defended and, on the North, already they were hard pushed by the ambitious and "turbulent Gallicks" who, aided by their Indian allies, raided the isolated farmhouses and small settlements. To expand towards the West, at least before a strong civilization had developed along the Atlantic coast line, was to multiply the existing dangers.

An entirely different conception appeared in what has been called the pioneer or the frontier spirit. A yearning for adventure, a temperamental restlessness, an inability to adapt oneself to the give and take of a strongly knit society, much more than a thirst for riches, impelled many other colonists to look beyond the mountains, to dream of new territories to explore and settle, of fertile land to till, of unlimited spaces and unlimited possibilities. Thus was established the psychological foundation of the theory of "Manifest Destiny" which appeared very early in the history of the country. It was a dynamic conception of America, clashing with the static conception of fixed boundaries. To use a modern illustration, the first one corresponded to a horizontal development such as can be observed in the expansion of Western cities; the other to a vertical growth typified by the skyline of New York City.

Already during the colonial period, the issue was clearly defined; the problem became more pressing after the colonies had decided to sever the bonds which made them part of the British Empire. Was America to develop along the same lines as the European nations, concentrating a large population on a relatively small territory? More precisely, were the people who called themselves Americans to accept the natural

boundaries which seemed to have been pre-established for them: on the North, Canada; on the West, the mountains. Were they justified, on the contrary, in entertaining the dream and the vision of an ever increasing population expanding westward, at least to the Mississippi and ultimately to the Pacific Coast? To the first group belonged such men as John Adams and those of the New Englanders whose eyes were turned towards the West Indies and Europe. To the second, Thomas Jefferson who, from his very childhood had been in close contact with the frontier, and, curiously enough, Benjamin Franklin.

We have been so accustomed to think of Franklin in terms of Old Richard's Almanach, as a world wise philosopher, slow and sure-footed, that we are apt to forget the adventurous lad who left Boston to explore the world, a restless, versatile, not easily curbed young man with a fervid imagination. As a point of fact, one should never speak of Franklin without keeping in mind at least the title of an old article of Paul Leicester Ford: "The Many-sided Franklin." It cannot be denied that he was a benign philosopher, the most European of all the Americans of his generation, the first American "bourgeois," a practical scientist and a bold theorist. But the grandfatherly "Papa Franklin" of the rue de Passy, the astute diplomat and the clear-minded patriot had worn in his youth the leather jacket of the hunter. In every American, even in our days, there is a "Westerner," and the most sedate ones have not been untouched by the pioneering spirit.

With the West, this Boston-born printer and newspaperman had a first hand acquaintance, which he would never have obtained if he had remained in New England. Philadelphia has gained a reputation for level-headedness, frugality and wisdom largely because the whole trend of life in the city seems to be colored by the traditional and not wholly legendary quaker and Franklinian characteristics. In the eighteenth century, however, it offered to the observer a curious mixture of advanced culture and pioneering ambitions. Very early and of necessity, the Philadelphians had to look westward, towards the mountains, and beyond the mountains, where began the rich region of the Ohio, the primeval wilderness and the limitless plains. There extended, at the back door of the

colony, an enormous territory where roamed the Indians, and in which the French had established outposts. To limit oneself voluntarily and deliberately is one thing; to feel that one's future development may be determined and hampered by the presence and threats of a none too friendly nation is quite another matter, which cannot be easily dismissed.

As early as 1751, Benjamin Franklin became keenly aware of that situation and took position as an advocate of a greater America. Putting down his "Observations concerning the increase of mankind, peopling of countries, etc.," he sketched out a grandiose picture of the development of the British colonies in North America:

> There is, in short, no Bound to the prolific Nature of Plants or Animals, but what is made by their crowding and interfering with each other's means of Subsistence. Was the Face of the Earth vacant of other Plants, it might be gradually sowed and overspread with one Kind only; as, for Instance, with Fennel; and were it empty of other Inhabitants, it might in a few Ages be replenish'd from one Nation only; as, for Instance, with *Englishmen*. Thus there are suppos'd to be now upwards of One Million *English* Souls in *North America*, (tho' 'tis thought scarce 80,000 have been brought over Sea,) and yet perhaps there is not one the fewer in *Britain*, but rather many more, on Account of the Employment the Colonies afford to Manufacturers at Home. This Million doubling, suppose but once in 25 years, will, in another Century, be more than the People of *England*, and the greatest Number of *Englishmen* will be on this Side of the Water. What an Accession of Power to the *British* Empire by Sea as well as by Land! What Increase of Trade and Navigation! What Numbers of Ships and Seamen! We have been here but little more than 100 years, and yet the Force of our Privateers in the late War, united, was greater, both in Men and Guns, than that of the whole *British* Navy in Queen *Elizabeth*'s Time. How important an Affair then to *Britain* is the present Treaty for settling the Bounds between her Colonies and the *French*, and how careful should she be to secure Room enough, since on the Room depends so much the Increase of her People. (Smyth ed. III, 71.)

There spake the seer and the prophet, but who could fail to recognize the pioneer in the concluding boast that "we are, as I may call it, *Scouring* our Planet, by *clearing America* of Woods, and so making this Side of our Globe reflect a brighter

Light to the Eyes of the Inhabitants in *Mars* or *Venus?*" Almost exactly a century later, in 1856, another American reviewing the work done by the pioneers and their epic battle against the forest, was to sing of the "lands of the make of the axe." The "many-sided" Franklin thus happens to be one of the unexpected but authentic predecessors of Walt Whitman.

In 1751, Franklin could only speak of the West by hearsay; in 1753, after travelling the rough trail to the newly settled village of Carlisle, as a member of the Commission which was to meet with representatives of the Six Nations, he came into direct contact with the backwood country and the American wilderness. It is well known that the famous plan adopted a year later by "the Commissioners assembled at Albany in July, 1754," was a direct result of the conclusions drawn by Franklin from his Western experience. The objects of the plan were twofold. The necessity of uniting the colonies for their mutual defence and security was emphasized and constituted the strongest appeal for a union; but, tucked away among the considerations, was an article recognizing that "The establishing of new colonies Westward on the Ohio and the Lakes, a matter of considerable importance to the increase of the British trade and power, to the breaking that of the French . . . would be best carried on by a joint union."

The chapter entitled "Indian Purchases" made it even plainer that the plan did not contemplate "freezing" the territorial possessions of the colonies as they were, and admitted that "several of the colony charters in America extend their bounds to the South Seas, which may be perhaps three or four thousand miles in length to one hundred or two hundred miles in breadth. . . ." Another chapter was frankly headed "New Settlements." It was proposed that 'the Grand Council' "make new settlements on such purchases, by granting lands in the King's name." To prevent the monopolization of the land by big purchasers, the grants were to be divided in small tracts. "Strong forts on the Lakes, the Ohio, etc., may at the same time they secure our present frontiers, serve to defend new colonies settled under their protection." Finally the whole scheme was revealed in the paragraph which read: "A particular colony has scarce

strength enough to extend itself by new settlements, at so great a distance from the old; but the joint force of the Union might suddenly establish a new colony or two in those parts or extend an old colony to particular passes, greatly to the security of our present frontiers, increase of trade and people, breaking off the French communication between Canada and Louisiana, and speedy settlement of the intermediate lands." The plan adopted by the Albany Convention was premature and came to naught, but Franklin found time to put down on paper very definite proposals "For settling two Western colonies in North America, with reasons for the plan." (Smyth, III, 358.) The "Plan" started with an almost lyrical description of the country to be settled:

> The great country back of the Appalachian Mountains, on both sides of the Ohio, and between that river and the Lakes is now well known, both to the English and French, to be one of the finest in North America, for the extreme richness and fertility of the land; the healthy temperature of the air, and mildness of the climate; the plenty of hunting, fishing and fowling; the facility of trade with the Indians; and the vast convenience of inland navigation or water-carriage by the Lakes and great rivers, many hundreds of miles around."

Then came the claim that this rich country which "must undoubtedly (perhaps in less than another century) become a populous and powerful dominion" was indispensable for the natural development of the British colonies: "Our people, being confined to the country between the sea and the mountains, cannot much more increase in number; people increasing in proportion to their room and means of subsistence."

Franklin's thesis was now complete. It had become a matter of life or death to prevent the junction between the French colonies of Louisiana and Canada, for such a junction would have "cut off" the British colonies from all commerce and alliance with the Western Indians and, at the same time, caused untold "inconveniences and mischiefs."

What he had proposed as a political philosopher, he even tried to put into effect as a soldier, in that curious episode of his career when he organized the militia and, without any military commission whatsoever, led them to the defense of Western Pennsylvania. But the main object remained the

settlement of the country west of the mountains. As he wrote to Peter Collinson, on June 20, 1755: "It is certain that People enough may be had, to make a strong English settlement or two in those parts. I wish to see it done, and I am almost indifferent how or by whom it is done. . . ."

So urgent was the need and so imperious the necessity, so magnificent the opportunity that he even contemplated the possibility of giving up the pleasant life of Philadelphia, his philosophical friends of the Junto and his books, to take up the life of the pioneer and establish a model colony. Whether or not it was more than a play of his fancy, one cannot doubt that in the letter he wrote to Rev. George Whitefield, on July 2, 1756, he expressed the secret desire of his heart:

> You mention your frequent wish that you were a Chaplain to an American Army. I sometimes wish that you and I were jointly employ'd by the Crown, to settle a Colony on the Ohio. I imagine we could do it effectually, and without putting the Nation to much expence. But I fear we shall never be called upon for such Service. What a glorious Thing it would be, to settle in that fine Country a large strong Body of Religious and Industrious People. What a Security to the other Colonies; and Advantage to Britain, by Increasing her People, Territory, Strength, and Commerce. Might it not greatly facilitate the Introduction of pure Religion among the Heathen, if we could, by such a Colony show them a better Sample of Christians than they commonly see in our Indian Traders, the most vicious and abandoned Wretches of our Nation? . . . Life, like a dramatic Piece should not only be conducted with Regularity, but methinks it should finish handsomely. Being now in the last Act, I begin to cast about for something fit to end with. Or if mine be more properly compar'd to an Epigram, as some of its few Lines are but barely tolerable, I am very desirous of concluding with a bright Point. In such an Enterprise I could spend the Remainder of Life with Pleasure; and I firmly believe God would bless us with Success, if we undertook it with a Sincere Regard to his Honour, the Service of our gracious King, and (which is the same thing) the Publick Good. (Smyth, III, 339.)

Little did he foresee at the time that, far from being "in the last act," the real play had not begun yet and that he would be called to be one of the main actors in a much greater drama, providing him with more "bright points" and a more "handsome finish" than he could have anticipated even in

his wildest dreams. As a first unexpected turn, instead of establishing his philosophico-religious Utopia on the banks of the Ohio, at the beginning of the following year, he was sent to London, to present the cause of Pennsylvania, and urge the adoption of measures to settle the frontier difficulties with the troublesome neighbors of the colonists.

While we cannot here dwell in detail on the many activities of Franklin in Europe, it may at least be recalled that, more than ever, he insisted on two main points, by him considered as absolutely essential if the colonies were to survive: security for the present, and possibilities of development for the future. He was still thinking of the American colonies as part of the British Empire, but in his imperialistic vision they were to play such a part as to decidedly eclipse the mother country. In his official communications, he observed a comparative reserve; in his private correspondence he threw all restraint to the winds and indulged in dreams of imperialistic grandeur. This appears in a most striking manner in the letter he wrote to Lord Kames to accompany a copy of his "Observations on the Peopling of Countries":

> No one can rejoice more than I do, on the reduction of Canada; and this is not merely as I am a colonist, but as I am a Briton. I have long been of the opinion, that the *foundations of the future grandeur and stability of the British empire lie in America:* and though, like other foundations, they are low and little seen, they are, nevertheless, broad and strong enough to support the greatest political structure human wisdom ever yet erected. I am therefore by no means for restoring Canada. If we keep it, all the country from the St. Lawrence to the Mississippi will in another century be filled with British people. Britain itself will become vastly more populous, by the immense increase of its commerce; the Atlantic will be covered with your trading ships; and your naval power, thence continually increasing, will extend your influence round the whole globe, and awe the world. If the French remain in Canada, they will continually harass our colonies by the Indians, and impede if not prevent their growth; your progress to greatness will at best be slow, and give room for many accidents that may for ever prevent it. But I refrain, for I see you begin to think my motions extravagant, and look upon them as the ravings of a mad prophet. (Smyth, IV, 4.)

The booklet itself contained the most complete exposition of the Franklinian doctrine of expansionism ever given by the author. The philosopher proved himself a very realistic politician when it came to justify in advance the annexation of Canada to the British Empire. The full title itself is sufficiently revealing: *The Interest of Great Britain Considered with regard to her Colonies and the acquisition of Canada and Guadaloupe to which are added Observations concerning the increase of Mankind, Peopling of Countries, &c.* It is more than the skillful plea of a very resourceful lawyer: throughout the whole argument is felt the deep conviction that the acquisition of Canada is a matter of life or death for the British colonies of America. It was of such importance that Franklin would not have hesitated a moment to give up all claims to the French possessions in the West Indies in order to obtain full "security" on the continent.

Nor did Franklin rest on his oars after the French had abandoned the main land and given up Canada and Louisiana. The problems of the "peopling" of the West remained opened, and to this task he gave his best efforts. It was at the same time to him a moral as well as a political problem. On the one hand, the Indians had to be persuaded to give up, for a consideration, a large part of their lands; but on the other hand, they had to be protected against the unscrupulous speculators and the brutal traders, adventurers and outlaws who lived on the fringe of the frontier. A complete expression of Franklin's views on the subject may be found in his *Remarks on the Plan for regulating the Indian Affairs*, written around 1766 at the request of Lord Shelburne. On September 12, of the same year, he wrote in the same vein to Sir William Johnson:

> I have long been of the Opinion that a well-conducted Western Colony, if it could be settled with the Approbation of the Indians, would be of great National Advantage with respect to the Trade and particularly useful to the old Colonies as a Security to their Frontiers. I am glad to find that you, whose knowledge of Indian affairs and the Temper of those People far exceeds mine, entertain the same sentiments, and think such an Establishment in the Illinois Country practicable. I shall not fail to use my best Endeavours here in promoting it, and obtaining for that purpose the necessary Grants.

But, immediately following that strictly realistic view of the situation comes a paragraph which shows that the humane philosopher had not yielded an inch:

> It grieves me to hear that our Frontier people are yet greater Barbarians than the Indians, and continue to murder them in time of Peace. I hope your Negotiations will prevent a new war, which those Murders give great Reason to apprehend; and that the several Governments will find some Method of preventing such horrid Outrages for the future.

Although not a specialist in Indian affairs and Indian psychology, he still remembered the negotiations conducted in 1753, in the village of Carlisle. He felt that it would not be fair to ask the Indians to change their customs to adopt ours. The moralist did not even attempt to conceal his sympathy and almost his admiration for these people who had not any prison system, no compulsion, knew no written contracts, and of whom it could be said that: "The Indian Trade, so far as Credit is concerned, has hitherto been carried on wholly upon Honour" (Smyth, IV, 470).

Neglecting no opportunity to advance what had now become with him a real mission, he has told us himself how, after a dinner with Lord Shelburne and Conway, he insisted again upon the desirability of establishing an outpost or rather settlement in the Illinois country. Apparently this was a very modest project, but the conclusion of the discussion indicated that he always kept in mind the possibility of a much more considerable expansion, for among the various advantages would be: "Furnishing provisions cheaper to the garrisons, securing the country, retaining the trade, raising the strength there which on occasion of a future war, might easily be poured down the Mississippi upon the lower country, and into the Bay of Mexico, to be used against Cuba or Mexico itself . . ." (Smyth, V, 46). This time the vision of a greater America extended not only beyond the Appalachian mountains but well into the Southwest.

Such dreams and such ambitions were not regarded with favor in London and were particularly objectionable to the Board of Trade. This became quite manifest in the controversy arising around the well-known "Walpole grant" which was the object of an adverse report written by Lord

Hillsborough. Leaving aside the details of the controversy, the proposition made by the noble Lord amounted to restricting the settlements to a region where they could be kept under control and would profitably conduct their commerce with England. This could not be done with settlements too far in the interior, and it was further declared that "The laying open of new tracts of fertile territory in moderate climates might lessen her [America's] present produce; for it is the passion of every man to be a landholder, and the people have a natural disposition to rove in search of good lands, however distant." (Smyth, V, 474.)

Franklin's answer to this argumentation should naturally be taken with a grain of salt. It is the plea of a very skillful advocate and not the dissertation of a political philosopher. Whether he believed it or not, he maintained that the new settlers would not have any tendency to become "a kind of separate and independent people." But his main point was, that nothing could prevent the colonies from expanding westward:

> To conclude: As it has been demonstrated, that neither royal nor provincial proclamations, nor the dread and horrors of a savage war, were sufficient, even *before* the country was purchased from the Indians, to prevent the settlement of the lands *over* the mountains, can it be conceived, that, *now* the country is purchased, and the people have seen the proprietors of Pennsylvania, who are the hereditary supporters of *British policy* in their own province, give every degree of encouragement to *settle* the lands *westward* of the mountains, the legislature of the province, at the same time, effectually corroborate the measure and several thousand families, in consequence thereof, settle in the *new county* of Bedford, that the inhabitants of the middle colonies will be *restrained* from cultivating the luxuriant country of the Ohio, joining to the *southern* line of Pennsylvania.

The note of defiance is sounded much more loudly, and the pioneer spirit taking pride in already conspicuous achievements manifests itself without any restraint in a piece printed in the *Gentleman's Magazine* for September 1773, under the title of "Rules by which a great empire may be reduced to a small one." "Those remote provinces have perhaps been acquired, purchased, or conquered, at the sole expence of the

settlers, or their ancestors, without the aid of the mother country," declared Franklin. Then came the enumeration of the results already obtained by this new and practically self-made people:

> In laying these taxes, never regard the heavy burdens, those remote people already undergo, in defending their own frontiers, supporting their own provincial governments, making new roads, building bridges, churches, and other public edifices, which in old countries have been done to your hands by your ancestors, but which occasion constant calls and demands on the course of a new people. (Smyth, VI, 131.)

Dr. Franklin still could proclaim himself a loyal Briton and remain attached to England intellectually and sentimentally; without perhaps fully realizing it, he had already become the spokesman of a new people and a greater America.

As we have seen, Franklin did not rely at any time on the support of Great Britain for the territorial development of the Colonies. Their expansion towards the West was a sort of natural force that no political power could stop or even control. The Independence did not change his views on the subject. In the draft of the Treaty of Alliance with France had been clearly expressed the firm purpose to exclude forever France from the American continent, and the clause had been accepted by the government of Louis XVI. In a brief outline, which he took along with him to France, Franklin had sketched tentative peace propositions in case an opportunity for negotiation should offer. One of the articles was quite in line with his previous expansionist views:

> To prevent those occasions of misunderstanding, which are apt to arise where the territories of different powers border on each other, through the bad conduct of the frontier inhabitants on both sides, Britain shall cede to the United States the provinces or colonies of Quebec, St. John's Nova Scotia, Bermuda, East and West Florida, and the Bahama Islands, with all their adjoining and intermediate territories now claimed by her. (Smyth, VI, 432.)

In the "motives," he even claimed that it was "absolutely necessary" for the United States to have these countries, "for our own security."

Through all the diplomatic negotiations which he conducted as American plenipotentiary in Europe, Franklin never departed from this view, even if finally he had to accept modifications when the final terms of the peace treaty were discussed.

Writing to d'Aranda, in April 1777, he maintains that in case Spain should conquer Pensacola, "the inhabitants of the United States shall have the free navigation of the Mississippi, and the use of the harbour of Pensacola." He is much more emphatic in a letter to John Jay, October 2, 1780: "Poor as we are, yet, as I know we shall be rich, I would rather agree with them to buy at a great price the whole of their right on the Mississippi, than sell a drop of its waters. A neighbour might as well ask me to sell my street door" (Smyth, VIII, 144). And again, writing to Robert Livingston, on April 12, 1782: "I see by the Newspapers that the Spaniards, having taken a little Post called St. Joseph, pretend to have made a Conquest of the Illinois Country. In what Light does this Proceeding appear to Congress? While they decline our offer'd Friendship, are they to be suffered to encroach on our Bounds, and shut us up within the Appalachian mountains?" (Smyth, VIII, 425.) On August, 12, of the same year, the same idea is again expressed almost in the same terms: "I will only mention that my conjecture of that court's design to coop us up within the Allegany Mountains is now manifested. I hope Congress will insist on the Mississippi as the boundary, and the free navigation of the river, from which they could entirely exclude us." (Smyth, VIII, 380.) If he feared Spanish encroachments, he was equally fearful of the British: "They wanted to bring their boundary down to the Ohio, and to settle their loyalists in the Illinois country. We did not choose such neighbours." (To Robert Livingston, Dec. 5, 1782. Smyth, VIII, 634.)

On all these points, the peace treaty was to give entire satisfaction to Franklin. Navigation on the Mississippi was to be free from its source to its mouth, and the river became the western boundary of the United States, at least for the time being.

Meanwhile Franklin was thinking not only of the im-

mediate issues to be settled, but of the future of the country. The territorial expansion of the United States was now secure; it remained to determine what kind of people were going to develop it. Here again the American philosopher had reached early very definite conclusions. The bulk of the population was to come from the natural increase of the people already established in the United States. Given room enough, the population would expand in close correspondence with the natural resources of the country. Immigrants would come, but immigration should be very carefully supervised, so as not to introduce undesirable elements. Newcomers from Europe would be welcome on two conditions only: if they were people of good character, and if they did not oppose an almost unsuperable resistance to the natural process of assimilation or, more exactly, Americanization. Such unfortunately was the case of the Germans already settled in Pennsylvania and for this reason, if for no other, German immigration was to be discouraged. The French would be acceptable, if they came only in small numbers, and such was the case of the colonists of Gallipolis who settled in the wilderness with the blessing of the old philosopher. But the most desirable element would remain in the future, as it had been in the past, the immigrants from the British islands who could be assimilated with a minimum of difficulty. Quite striking in this respect is the letter written to William Strahan from Passy, on August 19, 1784:

> The subject, however, leads me to another Thought, which is, that you do wrong to discourage the Emigration of Englishmen to America. In my piece on Population, I have proved, I think, that Emigration does not diminish, but Multiplies a Nation. You will not have fewer at home for those that go Abroad and as every Man who comes among us, and takes up a piece of Land, becomes a Citizen, and by our Constitution has a voice in Elections, and a share in the Government of the Country, why should you be against acquiring by this fair Means a Repossession of it, and leave it to be taken by Foreigners of all Nations and Languages, who by their Numbers may drown and stifle the English, which otherwise would probably become in the course of two centuries the most extensive Language in the World, the Spanish only excepted. It is a fact, that the Irish emigrants and their children are now in Possession

of the Government of Pennsylvania, by their Majority in the Assembly, as well as of a great Part of the Territory: and I remember well the first ship that brought any of them over. (Smyth, IX, 264.)

Two years earlier, in order to give of America "clearer and truer Notions of that part of the World than appear to have hitherto prevailed," he had written his "Information to those who would remove to America." In it, he had not failed to warn the prospective immigrants against exaggerated expectations. The famous passage in which he warned them, in his familiar humorous manner, has often been quoted:

> In short, America is the land of Labour, and by no means what the English call *Lubberland*, and the French *Pays de Cocagne*, where the streets are said to be pav'd with half-peck Loaves, the Houses til'd with Pancakes, and where the Fowls fly about ready roasted, crying, *Come eat me.*

But the rest of the picture must have been singularly attractive to ambitious people who, for generations, had lived in countries where every acre had been long pre-empted. To them, in spite of all warnings, America must have looked as a real Pays de Cocagne:

> Land being cheap in that Country, from the vast Forests still void of Inhabitants, and not likely to be occupied in an Age to come, insomuch that the Propriety of an hundred Acres of fertile Soil full of Wood may be obtained near the Frontiers, in many Places, for Eight or Ten Guineas, hearty young Labouring Men, who understand the Husbandry of Corn and Cattle, which is nearly the same in that Country as in Europe, may easily establish themselves there. A little Money sav'd of the good wages they receive there, while they work for others, enables them to buy the Land and begin their Plantation, in which they are assisted by the Good-Will of their Neighbours, and some Credit. Multitudes of poor People from England, Ireland, Scotland, and Germany, have by this means in a few years become wealthy Farmers, who, in their own Countries where all the Lands are fully occupied, and the Wages of Labour low, could never have emerged from the poor Condition in which they were born.

Nor was it to be feared that this condition might pass rapidly, and that America would ever experience the same crowding as the old world. There was place for everybody in the new world and particularly for persons "having a

number of Children to provide for, and desirous of bringing them up to Industry and to secure Estates for their Posterity." To make it more conclusive and more striking, instead of vague promises particular instances and figures were quoted:

> There small Capitals laid out in Lands, which daily become more valuable by the Increase of People, afford a solid Prospect of ample Fortunes thereafter for those Children. The Writer of this has known several Instances of large Tracts of Land, bought, on what was then the Frontier of Pensilvania, for Ten Pounds per hundred Acres, which after them, sold readily, without any Improvement made upon them, for three Pounds per Acre.

Already the word "frontier," as used here by Franklin, had taken a special American meaning. It did not express any more anything fixed, or a line to be modified only after hard-won battles or prolonged negotiations. The settlements of today were yesterday's frontiers and the frontiers of today would extend and expand almost indefinitely as America became more populous and as the American people advanced in their Westward march.

With this picture of the American frontier we may well bring our study to a close. From his early "Observations concerning the Increase of Mankind" of 1751 to the signing of the Peace Treaty with the British Commissioners, in 1782, Franklin had worked steadily and unswervingly to make his dream come true. No danger now remained of ever seeing the American people "cooped up within the Alleganys," or "cut off" from the rest of the continent.

There are many statues of Benjamin Franklin, but there is one yet to be erected that would fittingly call attention to one of the most outstanding achievements of a great American who was such a persistent and clearsighted prophet of a greater America. Somewhere, on the top of the ridge in the Alleghenys, on one of these mountain passes through which rode and tramped the pathetic line of the early settlers, we might well build a statue of Benjamin Franklin, not the arm-chair philosopher, but the young Postmaster General, the man whom the Moravians dubbed "General" Franklin, pointing towards the West, and looking over the rich land of Canaan whose gateways he had made accessible to his people.

DIFFERENTIATION WITH THE CINEMA INTEGRAPH.

BY

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§1. INTRODUCTION.

Recent developments in analysis by mechanical means, such as the Bush Differential Analyzer [1] and the Cinema Integraph,[2] have been in the field of the class of problems which can be reduced to a single or repeated integration. This is true of the solution of ordinary differential equations, and of the solution of integral equations to which the cinema integraph is particularly adapted. In the following we propose to describe a method by which the cinema integraph can be adapted to perform differentiation.[3] The method does not attempt to make a differentiator out of the cinema integraph, but rather to convert differentiation into a process which involves essentially integration, for which, of course, the cinema integraph is perfectly suited. This possibility grew out of the realization that the standard methods of numerical differentiation of a function $f(x)$ at $x = x_0$ consist essentially of the *summation* of the values of $f(x)$ in the neighborhood of x_0 after these values have been multiplied by suitable weighting factors. Thus, if $f(x)$ is given for a set of equidistant values x_n of interval θ and if we use central differences, the derivative

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[1] "The Differential Analyzer. A New Machine for Solving Differential Equations," V. Bush; J. FRANK. INST., **212**: 447, 1931.

[2] "The Cinema Integraph. A Machine for Evaluating a Parametric Product Integral," H. L. Hazen and G. S. Brown; J. FRANK. INST., **230**: 19, 1940.

[3] Other methods of mechanical differentiation have been proposed by F. W. Sears, *J. O. S. A.*, **25**: p. 162, 1935, and by H. S. Hele Shaw, *Phil. Trans. Roy. Soc. London*, **176**: p. 367, 1885.

at x_0 is given by [4]

$$\frac{df}{dx} = \frac{1}{\theta}\left[\frac{1}{2}(\Delta f_{-1} + \Delta f_0) - \frac{1}{3!2}(\Delta^3 f_{-2} + \Delta^3 f_{-1}) + \frac{4}{5!2}(\Delta^5 f_{-3} + \Delta^5 f_{-2}) - \cdots\right], \quad (1)$$

where the Δ's denote finite differences. As a first approximation we may consider only the terms in (1) involving first differences and write

$$\frac{df}{dx} = \frac{1}{\theta}\left[\tfrac{1}{2}(\Delta f_{-1} + \Delta f_0)\right] = \frac{1}{\theta}(\tfrac{1}{2}f_1 - \tfrac{1}{2}f_{-1}). \quad (2)$$

As a second approximation we may include the third differences in (1):

$$\frac{df}{dx} = \frac{1}{\theta}\left[\frac{1}{2}(\Delta f_{-1} + \Delta f_0) - \frac{1}{3!2}(\Delta^3 f_{-2} + \Delta^3 f_{-1})\right] = \frac{1}{\theta}\left(-\frac{1}{12}f_2 + \frac{2}{3}f_1 - \frac{2}{3}f_{-1} + \frac{1}{12}f_{-2}\right). \quad (3)$$

The third approximation would give

$$\frac{df}{dx} = \frac{1}{\theta}\left(\frac{1}{60}f_3 - \frac{3}{20}f_2 + \frac{3}{4}f_1 - \frac{3}{4}f_{-1} + \frac{3}{20}f_{-2} - \frac{1}{60}f_{-3}\right). \quad (4)$$

In (2) the derivative is obtained by multiplying f_1 by $\frac{1}{2}$, f_{-1} by $-\frac{1}{2}$, all other ordinates by zero, summing and dividing by θ. The error introduced by this approximation will be of the order of the third differences. In (3) and (4) the derivative is obtained by a similar process of weighting and summing, with the range of x over which the weighting factors are not zero, progressively enlarged. We are thus led to inquire whether we can find, instead of the above discontinuous weighting factors, a continuous weighting function, or functions, which when multiplied by $f(x)$ in the neighborhood of

[4] J. B. Scarborough, "Numerical Mathematical Analysis," p. 115, 1930, The Johns Hopkins Press, Baltimore.

x_0 and integrated, will yield an approximate value of the derivative of $f(x)$ at x_0.

§2. WEIGHTING FUNCTIONS FOR CENTRAL DIFFERENTIATION.

In attempting to derive continuous weighting functions corresponding to the discrete weighting factors in (2), (3) and (4) we can follow the same steps that lead to the derivation of (1). In deriving the latter equation one approximates to $f(x)$ in the neighborhood of x_0 by a polynomial of order n, which takes on the values f_n at the n points x_n. Expression (1) is then precisely the value of the derivative of the polynomial at x_0. In the continuous case, the approximation to $f(x)$ by a polynomial would correspond to the development of $f(x)$ into an infinite series of a suitable set of orthogonal functions and the value of the derivative of the series at x_0 would be taken for $\dot{f}(x_0)$. The computation of the coefficients of the series would involve, like in a Fourier analysis, integrations over $f(x)$, which could easily be carried out with the cinema integraph. The number of integrations would, strictly speaking, be infinite but presumably the contributions from the high order terms in the derivative of the series would be negligible, so that one could dispense with a finite number of coefficients. In fact, the approximation in the discrete case by a polynomial of order n would correspond exactly to the approximation in the continuous case by a series containing only n terms. Essentially the same results for the case of the use of Hermite orthogonal functions can be obtained from the following considerations.

Consider the integral

$$D_1(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2} \dot{f}(x + \theta z) dz. \tag{5}$$

As θ approaches zero, $D_1(x)$ approaches $\dot{f}(x)$. By partial integration we can transform $D_1(x)$ into

$$D_1(x) = \frac{2}{\theta\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2} z f(x + \theta z) dz$$

$$= \frac{1}{\theta\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-z^2} H_1(z) f(x + \theta z) dz, \tag{6}$$

provided we assume that for large z, $e^{-z^2}f(x + \theta z)$ vanishes. Here $H_1(z)$ denotes the Hermite polynomial of first order, the polynomial of order n being defined by

$$H_n(z) = (-)^n e^{z^2} \frac{d^n}{dz^n}(e^{-z^2}).^5 \tag{7}$$

Expression (6) gives a first approximation to $\dot{f}(x)$, which is equivalent to (2), the discrete weighting factors $\frac{1}{2}$ and $-\frac{1}{2}$ at $+\theta$ and $-\theta$ being now replaced by the weighting function

$$F_1(z) = \frac{2}{\sqrt{\pi}} z e^{-z^2}. \tag{8}$$

This function is tabulated in Table 1.

In order to estimate the error introduced by the above approximation, we expand $f(x + \theta z)$ under the integral sign in (6) around x into a Taylor series and integrate term by term, obtaining

$$D_1(x) = \dot{f}(x) + \frac{1}{4}\theta^2 f^{(3)}(x) + \frac{1}{32}\theta^4 f^{(5)}(x) + \cdots + \frac{\theta^{2n}}{4^n n!} f^{(2n+1)}(x) + \cdots. \tag{9}$$

As a further approximation, we attempt to modify the weighting function so as to remove the term in θ^2 on the right hand side of (9). For this purpose consider

$$D_3(x) = \frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-z^2}\dddot{f}(x + \theta z)dz$$

$$= -\frac{1}{\theta^3\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-z^2} f(x + \theta z)\frac{d^3}{dz^3}(e^{-z^2})dz$$

$$= \frac{1}{\theta^3\sqrt{\pi}}\int_{-\infty}^{\infty} e^{-z^2} H_3(z) f(x + \theta z)dz, \tag{10}$$

[5] It is unfortunate that there is as yet no accepted notation for Hermite polynomials. The notation of (7) is that given in R. Courant and D. Hilbert, "Methoden der Mathematischen Physik I," Julius Springer, Berlin, 1924, p. 76. We find that T. C. Fry in his book on "Probability," D. Van Nostrand Co., New York, 1928, p. 255, uses the notation

$$H_n(z) = (-)^n e^{z^2/2}\frac{d^n}{dz^n}(e^{-z^2/2}).$$

provided we assume that $z^3e^{-z^2}f(x + \theta z)$ vanishes at infinity. Clearly, $D_3(x)$ approaches $\dddot{f}(x)$ as θ approaches zero. For finite values of θ we find, by expanding $f(x + \theta z)$ under the

TABLE I.

Weighting Functions $F_1(z)$, $F_2(z)$, $F_3(z)$ for Central Differentiation and Weighting Functions $G_1(z)$, $G_2(z)$, $G_3(z)$ for Forward or Backward Differentiation.

(For a graphical representation of the Weighting Functions see Fig. 1.)

z.	$F_1(z)$.	$F_2(z)$.	$F_3(z)$.	$G_1(z)$.	$G_2(z)$.	$G_3(z)$.
0	0	0	0	−1.0000	−3.0000	−6.0000
0.05	0.0563	0.1406	0.2458	−0.9037	−2.6183	−5.0545
0.1	0.1117	0.2782	0.4849	−0.8144	−2.2711	−4.2116
0.2	0.2168	0.5334	0.9185	−0.6550	−1.6702	−2.7968
0.4	0.3846	0.9000	1.4723	−0.4022	−0.7776	−0.8366
0.6	0.4723	1.0108	1.5020	−0.2195	−0.1976	0.2898
0.8	0.4760	0.8854	1.1138	−0.0899	0.1618	0.8699
1.0	0.4151	0.6227	0.5708	0.0000	0.3679	1.1036
1.2	0.3208	0.3400	0.1192	0.0602	0.4699	1.1278
1.4	0.2225	0.1201	−0.1256	0.0986	0.5031	1.0337
1.6	0.1396	−0.0083	−0.1825	0.1211	0.4926	0.8819
1.8	0.0795	−0.0589	−0.1366	0.1322	0.4562	0.7101
2.0	0.0413	−0.0620	−0.0672	0.1353	0.4060	0.5413
2.2	0.0196	−0.0460	−0.0168	0.1330	0.3501	0.3883
2.4	0.0085	−0.0279	0.0068	0.1270	0.2939	0.2569
2.6	0.0036	−0.0145	0.0121	0.1188	0.2406	0.1491
2.8				0.1095	0.1922	0.0642
3.0				0.0995	0.1494	0.0000
3.2				0.0897	0.1125	−0.0463
3.4				0.0801	0.0814	−0.0777
3.6				0.0710	0.0557	−0.0971
3.8				0.0626	0.0349	−0.1070
4.0				0.0549	0.0183	−0.1099
4.2				0.0480	0.0054	−0.1076
4.4				0.0417	−0.0044	−0.1018
4.6				0.0362	−0.0117	−0.0936
4.8				0.0313	−0.0168	−0.0841
5.0				0.0270	−0.0202	−0.0741
5.2				0.0232	−0.0223	−0.0641
5.4				0.0199	−0.0233	−0.0544
5.6				0.0170	−0.0235	−0.0454
5.8				0.0145	−0.0231	−0.0371
6.0				0.0124	−0.0223	−0.0297
6.5				0.0083	−0.0192	−0.0151
7.0				0.0055	−0.0155	−0.0055
7.5				0.0036	−0.0120	+0.0031

integral sign in (10) into a Taylor series and integrating term by term, that

$$D_3(x) = \dddot{f}^{(3)}(x) + \tfrac{1}{4}\theta^2 f^{(5)}(x) + \cdots + \frac{\theta^{2n} f^{(2n+3)}(x)}{4^n n!} + \cdots. \quad (11)$$

If we now put

$$F_2(z) = \frac{1}{\sqrt{\pi}} e^{-z^2}[H_1(z) - \tfrac{1}{4}H_3(z)], \qquad (12)$$

we obtain from (6), (9) and (10)

$$\frac{1}{\theta}\int_{-\infty}^{\infty} F_2(z) f(x+\theta z)dz = f(x) - \frac{\theta^4}{32} f^{(5)}(x) - \frac{\theta^6}{192} f^{(7)}(x) - \cdots. \qquad (13)$$

A third approximation is obtained by using the weighting function

$$F_3(z) = \frac{1}{\sqrt{\pi}} e^{-z^2}[H_1(z) - \tfrac{1}{4}H_3(z) + \frac{1}{32} H_5(z)]. \qquad (14)$$

It will be seen from Table 1 that the weighting functions simulate the general trend of the weighting factors in (2), (3) and (4).

§3. WEIGHTING FUNCTIONS FOR FORWARD AND BACKWARD DIFFERENTIATION.

The weighting functions derived above are suitable for obtaining the derivative of $f(x)$ at x_0 when $f(x)$ is known both for values of the argument greater and less than x_0. The method can easily be extended to forward differentiation by limiting the range of integration from zero to infinity and using Laguerre instead of Hermite polynomials. Thus, one finds by expanding $f(x + \theta z)$ in the integrand into a Taylor series and integrating term by term, that

$$-\frac{1}{\theta}\int_0^{\infty} e^{-z} L_1(z) f(x+\theta z)dz = \frac{1}{\theta}\int_0^{\infty} e^{-z}(z-1) f(x+\theta z)dz = \dot{f}(x) + 2\theta\ddot{f}(x) + 3\theta^2\dddot{f}(x) + \cdots, \qquad (15)$$

$$\frac{1}{\theta^2}\int_0^{\infty} e^{-z} L_2(z) f(x+\theta z)dz = \frac{1}{\theta^2}\int_0^{\infty} e^{-z}(z^2 - 4z + 2) f(x+\theta z)dz = 2\ddot{f}(x) + 6\theta\dddot{f}(x) + 12\theta^2 f^{IV}(x) + \cdots, \qquad (16)$$

where $L_n(z)$ denotes the Laguerre polynomial of order n, defined by

$$L_n(z) = e^z \frac{d^n}{dz^n}(z^n e^{-z}). \tag{17}$$

By a procedure similar to the one used in the preceding section, one obtains for the successive weighting functions the expressions

$$G_1(z) = -e^{-z}L_1(z), \tag{18}$$

$$G_2(z) = -e^{-z}[L_1(z) + L_2(z)], \tag{19}$$

$$G_3(z) = -e^{-z}[L_1(z) + L_2(z) + \tfrac{1}{2}L_3(z)]. \tag{20}$$

These weighting functions can also be used for backward differentiation by merely writing $f(x - \theta z)$ for $f(x + \theta z)$ in the integrand. Clearly, one can also derive weighting functions for derivatives higher than the first, both for central differentiation and for forward or backward differentiation.

§4. PROCEDURE OF DIFFERENTIATION WITH THE CINEMA INTEGRAPH.

The cinema integraph[2] can be used to evaluate a parametric product integral of the form

$$R(x) = \int_a^{b \text{ or } x} P(x \pm z)Q(z)dz, \tag{21}$$

where P and Q are known functions and x is a parameter. With the cinema integraph the multiplication of the two functions and the integration of their product involves the use of radiation from a line source of light. The radiation passes through two apertures located successively along an axis perpendicular to the line source. Photographic films with transparent areas corresponding to the regions enclosed by the functions P and Q are placed in the respective apertures. One can show that the total radiation transmitted through the two successive function areas on the films is proportional to the integral being evaluated. The " positive " component of radiation and the " negative " component, arising respectively from the positive and negative parts of the integral, are separately collected by an optical system and impressed on two phototubes in a bridge circuit. By balancing this bridge

with radiation from a secondary source of light, the numerical value of the integral is obtained. Methods are available for calibrating the machine, and one reads the answer from a dial;

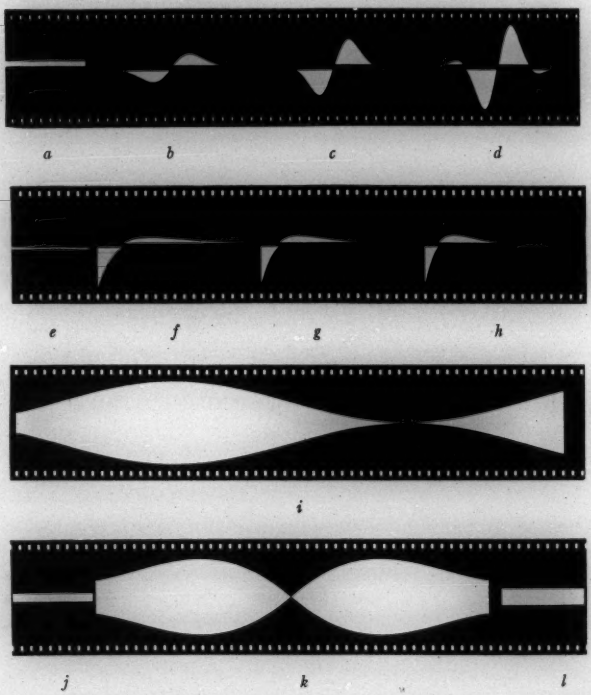

FIG. 1. Functions exposed on 70 mm. film. (*a*), (*e*), (*j*) and (*l*) are constants used for calibrating the cinema integraph; (*b*), (*c*) and (*d*) are the weighting functions F_1, F_2, F_3; (*f*), (*g*) and (*h*) are the weighting functions G_1, G_2 and G_3 (not to same scale); (*i*) is $(1+\sin x)$; and (*k*) is the function of Eq. (22*c*).

or when one function is of the form $P(x \pm z)$, the machine draws $R(x)$ on graph paper as a continuous function of the parameter x. These operations of the recording mechanism

are performed automatically by a small highspeed servo motor and a mechanical torque amplifier. A more complete description of the machine and its operation is given in reference 2.

Sections of the films for the weighting functions and the function f of Eq. (6) are shown in Fig. 1, as made from contact prints of the actual films. The base lines for the positive and negative areas of the weighting functions are slightly displaced from each other to permit the separation of positive and negative components of radiation by the optical system. The f functions are made all positive (or negative) by the addition of a constant, and the vertical height of the transparent area represents the value of the function. The rectangular areas are *constants* used to perform a simple integration for calibrating the machine.

To illustrate differentiation with the cinema integraph, f in Eq. (6) was taken as a sine function. The infinite limits were approximated by the limits -3 to $+3$ and a value $\theta = \frac{1}{3}$ was arbitrarily chosen. The period of the sine function was taken as 12 units of the variable of integration z. Therefore the derivative of $\sin x$ is given by

$$\frac{d}{dx}(\sin x) \doteqdot \frac{6}{\pi}\left\{\frac{2\cdot 3}{\sqrt{\pi}}\int_{-3}^{3} e^{-z^2} \times z\left[1 + \sin\frac{\pi}{6}\left(x + \frac{z}{3}\right)\right]dz\right\}, \quad (6a)$$

where unity is added to the sine to make f a positive valued function (see Fig. 1*i*). The derivative of $\sin x$, as drawn by the cinema integraph from the evaluation of Eq. (6a), is given in Fig. 2. Exact values are plotted as small circles. The largest error occurs when the argument of the sine is 45°, and is about 8 per cent. of the maximum value of the derivative. The error is much less for most of the derivative.

The accuracy of the cinema integraph is between 0.5 per cent. and 1 per cent., or less, of the integral of the *absolute values* of the functions in the integrand. In Eq. (6a) the maximum value of the derivative is unity, and the maximum of the integral of absolute values is about 13. The accuracy of the cinema integraph must be based upon the latter value, and accordingly the largest machine error is 8/13 per cent.

As a check on the accuracy of the method independent of the cinema integraph, Eq. (6a) was evaluated for $x = 0$ by an accurate graphical method and found to be 0.993 in comparison with the exact value of unity.

The accuracy of the differentiation can be improved by using a smaller value for θ. The cinema integraph accuracy can be improved by separating the function f into two parts. One part should be a function which can be differentiated exactly and the other should be preferably a small part for

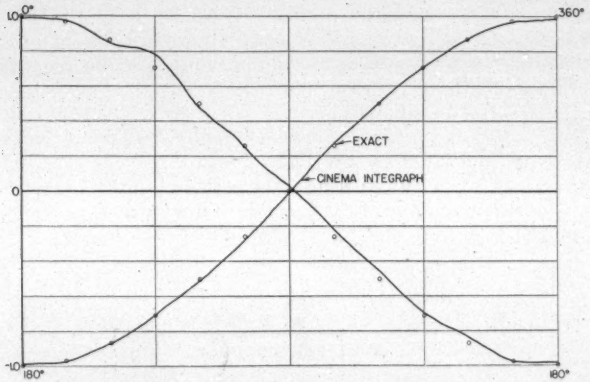

FIG. 2. Derivative of sin x by Eq. (6a).

the cinema integraph to differentiate. Then the derivative of f is the sum of the derivatives of the two parts. This scheme is illustrated by separating sin x into

(a) a linear function through the points (0°, 0) and (90°, 1) for $-30° \leq x \leq 120°$;
(b) a linear function through the points (90°, 1) and (180°, 0) for $60° \leq x \leq 210°$;
(c) the function representing the difference between sin x, and (a) and (b) to which a constant is added to give a positive function (Fig. 1k).

(22)

The function (c) is differentiated with the cinema integraph by Eq. (6). Since the derivatives of (a) and (b) are constants, the zero of the output table of the cinema integraph is shifted appropriately and the machine draws the derivative of f as a continuous function. The curve of Fig. 3 is the derivative of $\sin x$ as obtained in this manner. Similarly, the method was used with two linear segments and a parabola as the parts to be differentiated exactly. The difference between the exact derivative of $\sin x$ and the calculated derivative was too small

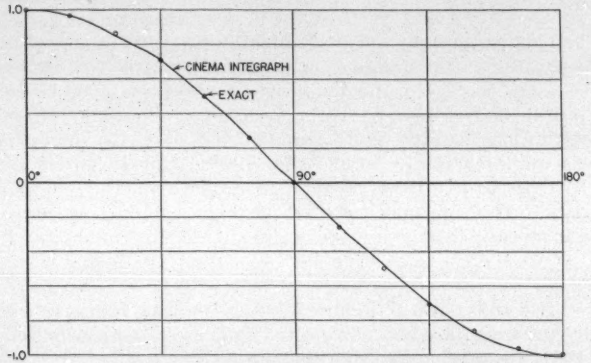

FIG. 3. Derivative of $\sin x$ by use of Eq. (22).

to show on a graphical plot. The maximum error was 0.6 per cent. and the average error 0.3 per cent.

The weighting functions of Eqs. (12) and (14) were used to differentiate $\sin x$ by the methods described above. The results were of the same order of accuracy. With these latter weighting functions small variations in the transparency of the film for f caused slightly more irregular derivative curves. The weighting function to be used in a particular problem should be selected on the basis of what is known or can be estimated about the derivative of the function being considered. The weighting function $F_1(z)$ of Eq. (10) has some advantage with the cinema integraph in averaging small errors of the function films.

The weighting functions of Eqs. (18), (19) and (20) for forward differentiation were tried by approximating the $(0, \infty)$ limits of integration by $(0, 6)$ limits. Since e^{-z} in these weighting functions approaches zero less rapidly than e^{-z^2} in the former weighting functions, a greater range of integration is required for satisfactory accuracy. A practical difficulty arises with these functions in preparing the film, due to the relatively high peak in the functions at the origin (see Fig. 1). The peak limits the scale and permits only a small transparent area for transmission of radiation. The difficulty can be avoided by placing parts of the peak on separate sections of film and performing the integrations in parts. The sum of the separate parts would give the required derivative.

The accuracy of the derivative of a function given by empirical data is limited by the accuracy of the data. If full advantage is taken of the method described for increasing cinema-integraph accuracy, the error in the derivative will be three to five times as great as the error in the empirical data. This presupposes that the higher derivatives either vanish or their effect is made negligibly small by the choice of θ.

APPENDIX.

It is of interest to inquire as to the limiting form assumed by the weighting function $F_n(z)$ as n is increased indefinitely. To do this we have to sum the infinite series

$$F(z) = \frac{1}{\sqrt{\pi}} e^{-z^2} \Big[H_1(z) - \tfrac{1}{4} H_3(z) + \cdots + \frac{(-)^n}{4^n n!} H_{2n+1}(z) + \cdots \Big]. \quad (23)$$

Now from the generating function of Hermite polynomials

$$e^{-t^2+2ty} = \sum_{n=0}^{\infty} \frac{H_n(y)}{n!} t^n,$$

we obtain

$$e^{-t^2} \sinh 2ty = tH_1(y) + \frac{t^3}{3!} H_3(y) + \cdots + \frac{t^{2n+1}}{(2n+1)!} H_{2n+1}(y) + \cdots. \quad (24)$$

By putting $t = i\tau$, (24) is transformed into

$$e^{\tau^2} \sin 2\tau y = \tau H_1(y) - \frac{\tau^3}{3!} H_3(y) + \cdots + \frac{(-)^n \tau^{2n+1}}{(2n+1)!} H_{2n+1}(y) + \cdots. \quad (25)$$

Now multiply both sides of (25) by $\tau e^{-a^2\tau^2}$, where $b^2 = a^2 - 1$ is positive, and integrate from zero to infinity:

$$\int_0^\infty e^{-b^2\tau^2} \sin(2\tau y)\tau d\tau = \frac{\sqrt{\pi}}{2b^3} y e^{-y^2/b^2}$$

$$= \frac{\sqrt{\pi}}{4a^3}\left[H_1(y) - \frac{1}{4a^2} H_3(y) + \cdots + \frac{(-)^n}{(4a^2)^n n!} H_{2n+1}(y) + \cdots \right]. \quad (26)$$

Hence

$$F(z) = \operatorname*{Lim}_{b\to 0} \frac{2}{\sqrt{\pi}} \frac{(b^2+1)^{\frac{1}{2}}}{b^3} z e^{-z^2 - z^2/b^2}, \quad (27)$$

$$\dot{f}(x) = \frac{1}{\theta}\int_{-\infty}^{\infty} F(z) f(x+\theta z) dz$$

$$= \operatorname*{Lim}_{b\to 0} \frac{2(b^2+1)^{\frac{1}{2}}}{\sqrt{\pi}\theta b^3} \int_{-\infty}^{\infty} f(x+\theta z) e^{-z^2 - z^2/b^2} z dz$$

$$= \operatorname*{Lim}_{h\to 0} \frac{2(h^2/\theta^2+1)^{\frac{1}{2}}}{\sqrt{\pi}h} \int_{-\infty}^{\infty} e^{-z^2 - h^2 z^2/\theta^2} f(x+hz) z dz$$

$$= \operatorname*{Lim}_{h\to 0} \frac{2}{\sqrt{\pi}h} \int_{-\infty}^{\infty} e^{-z^2} f(x+hz) z dz$$

$$= \operatorname*{Lim}_{\theta\to 0} \frac{1}{\theta\sqrt{\pi}} \int_{-\infty}^{\infty} F_1(z) f(x+z) dz. \quad (28)$$

The best approximation to the derivative is therefore obtained by using $F_1(z)$ for the weighting function and reducing the θ-scale to the smallest practicable value.

Construction Men Study New Ways to Engineer Timber.—Engineers employed by the various construction companies in and near Seattle, Washington, are going back to school to learn the latest methods of timber engineering. Their professors are men who have had specialized experience in the newly developed techniques of engineering in timber developed by the growing acceptance of the Teco Connector System of timber construction. The "course" is the idea of Chester Hogue and T. K. May, both of West Coast Lumbermen's Association, and J. B. Dodge of the Northwest Bolt and Nut Company. Nearly 40 engineers attended the first meeting of the "class" at the U. S. Engineers Office, Seattle, and according to Professor Dodge they have since been plying him with questions on how to apply the Teco Connector System to their specific construction problems. Between 40 and 50 engineers from the Siems Drake Spokane Company, the Puget Sound Bridge and Dredging Company, and the Associated Shipbuilders attended the second night school session.

R. H. O.

Research on Nickel at Mellon Institute.*—The International Nickel Company, Inc., New York, N. Y., has resumed investigational work at Mellon Institute of Industrial Research, Pittsburgh, Pa., by the establishment of an Industrial Fellowship that began operation on July 1. This Fellowship is concerned with a comprehensive program of scientific research on certain problems in the chemistry and technology of nickel, embracing particular attention to the preparation, properties, and uses of nickel compounds, especially organic derivatives.

Dr. John Gilbert Dean, a physico-organic chemist, is the incumbent of this Fellowship. He received his professional education at Brown University (Ph.B., 1931; M.S., 1932) and Columbia University (Ph.D., 1936) and has subsequently had five years of experience in applied chemical research.

* Communicated by the Director.

PHOTOGRAPHY AND THE FRANKLIN INSTITUTE.*

BY

W. N. JENNINGS.

Early one morning, sixty years ago, I started out to make street scene snapshots of old Philadelphia. Upon turning down Seventh Street, below Market, I came across an ancient marble structure set back from the building line. Curiosity led me to walk up the well-worn marble steps, between Ionic Columns, and enter the "Actuary's Office" on the first floor, where a portly, smiling gentleman greeted me. He might have just stepped out of a volume of Dickens.

"David Holman, at your service, sir," said he, allowing his steel-rimmed spectacles to slide down his bald head, fronting a pair of twinkling gray eyes. I explained that I was a stranger in Philadelphia and would like to know more about the sign, "Franklin Institute," on the front doorway.

"Surely, with pleasure, sir," he replied. "This old Institute is a Memorial to Benjamin Franklin and contains some interesting Franklin relics. If you will follow me, I will be glad to show you through the building.

"In the Reading Room upstairs we will find the original Electrical Machine constructed by the great Philosopher and used to entertain his many friends in his Philadelphia home. Here are his work-bench and inking stone, together with a few 'First Editions.'" And as we wandered over the old Institution for over an hour, a large placard bearing the word "SILENCE" seemed to be superfluous, for the place was as silent as the tomb of old King "Tut."

Next I was introduced to the Secretary, Dr. W. H. Wahl, another Dickens character, who showed me many interesting manuscripts, and before I left the building I had in my pocket a Membership ticket, little dreaming that The Franklin Institute would thereafter take such a large slice out of my life, and bring me so many life-long friends.

* The illustrations for this article were supplied through the courtesy of the American Museum of Photography, 338 South 15th Street, Philadelphia, Pa.

Next morning, with my camera, I followed in the footsteps of the great Philadelphian, snapshotting The Franklin Institute; Independence Hall, where the old Statesman signed the Declaration of Independence; Second Street Market, where he purchased his pennyworth of bread, and carried two loaves under his arms, to the intense amusement of the girl he afterward married; then the brick house with marble steps and green shutters where Franklin first slept in Philadelphia; and, nearby, I photographed the "Betsy Ross House." Then, having snapshotted the now busy site of the meadow at 9th and Filbert Streets where, rumor has it, Franklin and his son sneaked away to fly a kite, capture a flash of lightning, and win Immortality, I rounded up my delightful morning's walk at 5th and Arch Streets where lies the dust of the great Philosopher and his faithful Deborah.

* * * * * * * *

When I showed Doctor Wahl these photographs he was greatly interested. "The lectures at our Monthly Meetings," said he, "are usually highly technical and poorly attended. This morning I was wondering how this could be remedied, and here is the answer: Turn these photographs into lantern slides, and let the next Monthly Lecture have this appended notice:

"AT THE CONCLUSION OF THE ABOVE LECTURE
W. N. JENNINGS WILL SHOW A NUMBER OF
LANTERN SLIDES: 'IN THE FOOTSTEPS OF
BENJAMIN FRANKLIN.'"

As a result, with the old Lecture Hall crowded to capacity, this became a regular feature of the Monthly Meeting under such titles as

"Street Scenes in Colonial Philadelphia"
"In Darkest Africa" (9th and Lombard Streets)
"Chinatown"
"Street Markets of old Philadelphia"
"Colonial Germantown"
"Under the Hat Brim of 'Billy Penn'"
&c., &c.

Then came quite a number of "Philadelphia Firsts" (original photographs shown for the first time at The Franklin Institute), a feature that never failed to fill the Lecture Hall; and now that there is a growing interest in amateur photography, and we can easily prepare and project lantern slides in natural color, I would strongly recommend this method of popularizing the monthly lectures, and thus increasing membership.

* * * * * * * *

It was exactly sixty years ago that I caught my first "Jove's Autograph" after many nocturnal exposures, to show

FIG. I. One of the lightning photographs made by W. N. Jennings.

artists they were wrong in their delineation of a flash of lightning. This was shown for the first time at The Franklin Institute. I followed this "hobby" for over ten years and made many scientific friends here and abroad. Among these

was John Tyndall, who wrote me many autographic letters and lent encouragement to a then difficult task. In his last communication, just before his untimely death, he says, "I hope you have carefully filed all your lightning photographs, as in the end they will form a collection of the highest importance." My friend, Dr. MacEachron, of the General Electric Company, by his brilliant lightning experiments from the top of the Empire State Building, New York, has made this observation prophetic. Since the award to me of the Wetherill Medal (1930) I have donated all my lightning material to the Library of The Franklin Institute.

* * * * * * *

The first "Movie" was brought to my attention by Mr. Henry Heyl, a member of the Board of Directors of The Franklin Institute, when he asked me to make and mount a number of small transparencies, which, placed intermittently upon the rim of a round glass plate and rapidly turned, produced the illusion of motion. This was first shown by Mr. Heyl at the Academy of Music, Philadelphia, and afterward at The Franklin Institute on March 16, 1870.

Among the pioneers in motion photography was Eadweard Muybridge, who lectured at the Institute in February, 1883, upon "Attitudes of Animals in Motion." By invitation I visited his studio at the University of Pennsylvania and watched him operate electrically his sixteen cameras. Muybridge afterward undertook a monumental work on motion photography and carried it to completion. One large volume is now in the Library of The Franklin Institute. This clever, tireless worker suffered the fate of "The Man Before His Time." He surely deserved much greater recognition.

To C. Francis Jenkins, of Washington, D. C., goes the credit of being the original inventor of the Motion Picture Camera and Projector. He lectured with the machine at the Institute on December 18, 1895, and his "Phantoscope" won him the Elliott Cresson Gold Medal (1897).

Gustav Dietz, of New York, invented a very clever photographic Shutter and was invited to demonstrate it at the Institute. By an entirely new principle the four blades of this shutter could be operated at 2000th part of a second with-

out jarring the lens. The inventor lectured before the Institute on October 5, 1911, on "High Speed Photography," the shutter being for the first time synchronized with a flashlight, while an athlete wearing "shorts" leaped over three sugar barrels. While I was developing the plate in a side room, the lecturer was preparing another exposure. I immersed the lantern slide plate in strong developer, rinsed it in water, soaked it in formaldehyde, and handed it to the lantern operator, and a loaded plate-holder to Mr. Dietz.

The picture appeared upon the lantern screen, clear as crystal, for about a minute, then slowly faded away. In this way we made a number of high-speed flashlights (including a live chicken thrown up in the air) and wound up the lecture with a group flashlight of the audience applauding the Lecturer. For the invention of the "Multi-speed Shutter" Dietz was awarded the John Scott Medal (1909). It is now of great service in aerophotography.

Another very interesting Franklin Institute highlight was a Lecture by Nikola Tesla when he transmitted electric light and power without wires. This was presented on February 2, 1893, under the title "An Experimental Study of Light Effects Produced by Alternating High Potentials." He was a fascinating Lecturer with a keen sense of humor.

"See this boy's top on that plate," said he. "Notice that it is 'dead' and not connected with wires? Now, at a snap of my finger, it wakes up and begins to spin. See? Faster and faster, until it fairly hums. Do you know what that means? Someday we shall transmit power by wireless from Niagara Falls to this building, and by the same agency we shall send the tones of the human voice around the world. I see my friend, Professor Houston, sitting at the back of the hall, and will ask him if he will be so good as to unscrew an electric light bulb overhead and bring it to me? Thank you, Professor. Now, at your approach we will watch the bulb. Notice, no wire connection. Hold it up so that we can all see it. See, it begins to glow. Brighter and brighter. Now hand the brightly lighted bulb to me. Note how the top on the table is spinning. It is really a little motor synchronized with a special dynamo in the basement, of such high frequency,

that electric energy passes over and does not penetrate the nervous system.

"'But, you ask, suppose this dynamo breaks down. What will happen?'

"Professor Houston at that moment will fall dead," was Tesla's reply, followed by a gale of laughter. "But," he continued, "the joke would not alone be on the Professor, for, at that same moment every person in this hall would also drop dead. No, you need not get up and leave, my friends,

FIG. II. Air View over Philadelphia. Taken July 4, 1893, by W. N. Jennings. This is one of the four views taken on this first successful photographic excursion in a free balloon.

for you may be sure that dynamo will not stop during this demonstration." Upon this occasion Tesla forecasted many a wonder of Radio, which have since become commonplace.

After watching "Professor" King make several balloon ascensions in Fairmount Park, I got the balloon fever, built myself a fixed focus camera and interviewed the genial aeronaut, making a bid for a place in his balloon basket.

"If you want to throw away fifty dollars on a balloon flight with your camera, it is up to you," he said; "but I warn

you that it will result in failure for the following reasons: First, in summertime there is generally a blue haze between lens and landscape. Second, gas escaping from the open neck of the balloon is apt to leak in your camera and fog your plates. Third, to hold and operate that big camera in mid-air for the first time will result in photographic failure."

But on a windy fourth of July, 1892, I took my place in

FIG. III. The party of eight (photographed inside balloon) who went up in the first flight (1907) of the "Benjamin Franklin" balloon. W. N. Jennings is on extreme left.

the balloon basket, ready to go up, when the balloon burst just as we were leaving the ground. This gave me an opportunity to construct and test a new aero-camera with gas-tight bellows and plate-holders, and the lens fitted with a light yellow color filter to cut out blue haze; the plate-holders being loaded with orthochromatic plates. Then, on the evening of July 4th, 1893, we sailed skyward; when I succeeded in making several needle-sharp photographs, clear as crystal. Thirty by

forty enlargements from these negatives were first shown at The Franklin Institute, the World's Fair, and Crystal Palace, London, where they produced quite a sensation.

With my neighbor, associate and life-long friend, Frederic E. Ives, the inventor of half-tone engraving, now in world-wide use, and the Ives system of color photography, in 1891–1894, I made two excursions to the Yellowstone Park, resulting in a Lecture at The Franklin Institute:

"F. E. IVES AND W. N. JENNINGS
WILL LECTURE AT THE FRANKLIN INSTITUTE
ON THE YELLOWSTONE PARK WITH COLOR CAMERA
AND KODAK."

when for the first time photographs in natural color were shown on the lantern screen, and so successful was this demonstration that, by request, the lecture was repeated three times, and Mr. Ives was awarded an Elliott Cresson Medal (1893) for his invention of tri-chromatic photography. He afterward took out over seventy patents and we were constant visitors to the old institution.

Then, for two years I was associated with Mr. Ives in London, Berlin and Paris, developing Color Photography; returning to resume my membership in the Institute and the Committee on Science and the Arts, of which body I am its oldest living member.

Since that time I have shown at the Institute about twenty "Franklin Institute Firsts" and have written several articles for the JOURNAL of The Franklin Institute.

So now, when you walk down Seventh Street, below Market, you will see the little ancient marble building that has escaped the profane hands of the "Building Wrecker," owing to the generosity of A. Atwater Kent, a building that for all time will perpetuate the name and fame of Benjamin Franklin; and if you will continue your footsteps along the Parkway to Twentieth, you will see where a Dream has come true, by the erection of a splendid Memorial building to Benjamin Franklin and "Poor Richard," and the finest Scientific Institution in the world.

DAMPER CIRCUITS AND ROTOR LEAKAGE IN THE TRANSIENT PERFORMANCE OF SATURATED SYNCHRONOUS MACHINES.

BY

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1. INTRODUCTION.

Several important phenomena in the operation of electric power systems are due to the action of the stator and rotor leakage of the feeding synchronous generators and particularly to the linkage of the rotor leakage fluxes with the damper circuits in the poles of the machines. In a previous paper,[1] these effects in saturated synchronous machines were treated by a generalized method. In this paper we shall consider the interaction of rotor leakage and damper circuits in greater detail, with full regard to the magnetic saturation of the main flux, and we shall include the reaction of the armature winding on the damper circuits. This detailed analysis may be justified by the fact that the development of the dangerous short-circuit currents in both magnitude and time, as well as the development of the recovery voltage in circuit breakers, limiting the interruption of short-circuit currents, which constitute two of the dominating problems in electric power engineering, depend preponderantly on these very effects.

2. FUNDAMENTAL CONDITIONS OF THE ELECTRIC AND MAGNETIC CIRCUITS.

We consider the main circuits of a synchronous machine as in Fig. 1, and shown in more detail in the cross-section of Fig. 2. Actually, the various magnetic fields overlap and influence one another, but for the sake of simplicity we assume them to be separated into main and leakage fluxes, as is common in the theory of machines.

In the *stator circuit*, the resistance of which we may neglect,

[1] R. Rüdenberg, "Saturated Synchronous Machines under Transient Conditions in the Pole Axis," *Transactions A.I.E.E.*, 1942, Vol. 61, p. 297, contains a list of references.

the internal electromotive force E is given by the terminal voltage V and the leakage voltages caused by the stator and rotor leakage fluxes Φ_s and Φ_r,

$$E = V + \omega N_a \Phi_s + \omega N_a \Phi_r, \qquad (1)$$

ω being the constant angular frequency and N_a the effective number of turns of the stator winding. If the terminals are loaded by a current I in a constant impedance X, the first

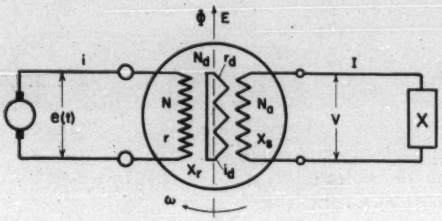


FIG. 1. Equivalent electric circuits of a synchronous machine.

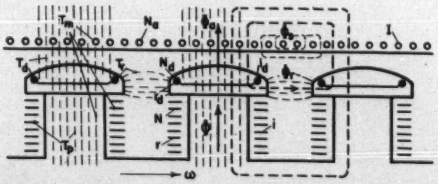


FIG. 2. Linkage of magnetic and electric circuits in the cross-section of a synchronous machine.

term on the right-hand side of equation (1) is XI. We consider primarily the case where X is a pure reactance, so that equation (1) remains algebraic. Iron losses may be neglected. If the stator leakage flux is proportional to the current, we can express the second term on the right-hand side of equation (1) by $x_s I$ where x_s is the stator leakage reactance. The flux of the third term, however, is linked, according to Fig. 2, not only with the stator currents but also with the damper currents and therefore this term cannot be expressed so simply.

The *excitation circuit* of the rotor is fed by an external voltage e which is given either as constant or as a known function of time. With resistance r, number of field turns N, and rotor pole flux Φ, the exciting current i is determined by

$$e(t) = ri + N\frac{d\Phi}{dt}, \tag{2}$$

the last term giving the influence of the change of flux and being predominant during the transient state.

In the *main magnetic circuit* of Fig. 2, the excitation current i produces the magnetic flux Φ in the pole core, which requires a magnetizing current i_μ dependent on Φ and thus on the

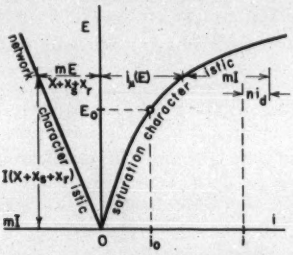


FIG. 3. Constant magnetic and electric characteristics of a synchronous machine in transient state.

corresponding electromotive force E according to the magnetic saturation curve shown in Fig. 3. In addition, the excitation current balances the armature reaction of the load current I, which may be purely reactive, and also the reaction of the damper current i_d, flowing in the number of turns N_d. Hence the excitation of the main flux is determined by the relation

$$i = i_\mu(E) + \frac{N_a}{N} I - \frac{N_d}{N} i_d. \tag{3}$$

In this equation we count as positive: a magnetizing damper current i_d, but a demagnetizing inductive stator current I.

The *damper circuit* according to Fig. 2 is linked completely with the armature flux Φ_a, this being the difference of the rotor

core flux Φ and the rotor leakage flux Φ_r. Therefore the current in the short-circuited damper with resistance r_d is given by

$$0 = N_d \frac{d}{dt}(\Phi - \Phi_r) + r_d i_d. \tag{4}$$

In the *rotor leakage circuit* the flux Φ_r can be considered, as is shown in Fig. 2, as excited by the combined effect of stator current and damper current, these two having opposite positive direction. The effective rotor leakage flux is therefore

$$\Phi_r = \frac{x_r}{\omega N_a}\left(I - \frac{N_d}{N_a} i_d\right), \tag{5}$$

if we define the rotor leakage reactance at the steady state, related to the stator current and without the influence of the damper current, as

$$x_r = \omega \frac{N_a \Phi_{r0}}{I_0}. \tag{6}$$

The rotor pole flux Φ and the e.m.f. E are always proportional,

$$\frac{\Phi}{E} = \frac{\Phi_0}{E_0}, \tag{7}$$

if Φ_0 and E_0 denote the rated values of flux and stator voltage in the machine at no load. With rated rotor voltage e_0 the last term of equation (2) therefore can be written in either of the two following forms:

$$N \frac{d\Phi}{dt} = T_m \frac{dE}{dt} = e_0 T_p \frac{d(E/E_0)}{dt}, \tag{8}$$

where

$$T_m = \frac{N\Phi_0}{E_0} \tag{9}$$

is a time constant of the complete machine which is entirely independent of the magnetic saturation, while

$$T_p = \frac{N\Phi_0}{e_0} \tag{10}$$

is a time constant of the rotor poles with their field windings

only, which is constant so long as the rated rotor values remain unchanged.

For machines of the usual design the numerical value of this time constant is of the order of T_p = 3–6–12 sec. for generated powers of 100–1000–10,000 kva. per pole of the synchronous machine.

3. SOLUTION OF THE CIRCUIT EQUATIONS.

Equations (1) to (5) constitute the fundamental equations of our problem which shall be solved for the transient state and with variable magnetic saturation of the machine. Although it is not absolutely necessary, we assume, for the sake of simplicity, that the important saturation of the main magnetic circuit is concentrated at the pole cores, where it has maximum effect, and we also assume that the saturation of the magnetic leakage circuits is only low. In the given set of equations we eliminate the subordinate magnitudes, particularly those of linear dependence, and we relate the remaining terms to those parameters which contain the saturation of the main magnetic circuit, particularly to the magnetic no-load characteristic $i_\mu(E)$.

If we substitute in equation (2) the flux given by equation (8) and insert in equation (2) the value of the exciting current i of equation (3), we obtain the main equation for the change with time of the e.m.f. of the machine

$$T_m \frac{dE}{dt} = e(t) - r[i_\mu(E) + mI - ni_d], \qquad (11)$$

where

$$m = \frac{N_a}{N}, \qquad n = \frac{N_d}{N} \qquad (12)$$

express the turn ratios of armature and damper windings to the field coils. Equation (11) could be solved if the content of the square bracket were dependent only on the e.m.f. E, as is the no-load magnetizing current i_μ.

By use of equation (5), however, and by expressing voltage and fluxes in equation (1) by reactances and currents, we obtain

$$E = XI + x_s I + x_r \left(I - \frac{N_d}{N_a} i_d \right), \qquad (13)$$

showing that the rotor leakage drop in the transient state depends upon the value of the damper current. Hence the stator current, expressed by e.m.f. and damper current, is

$$I = \frac{E}{X + x_s + x_r} + \frac{x_r}{X + x_s + x_r} \frac{n}{m} i_d, \qquad (14)$$

and the last two terms in the bracket of equation (11) become

$$mI - ni_d = \frac{mE}{X + x_s + x_r} - \frac{X + x_s}{X + x_s + x_r} ni_d. \qquad (15)$$

Thus we have substituted the e.m.f. E for the stator current on the right-hand side of equation (11), E being the main variable of this differential equation, and we obtain

$$T_m \frac{dE}{dt} = e(t) - r\left[i_s(E) + \frac{mE}{X + x_s + x_r} - \frac{X + x_s}{X + x_s + x_r} ni_d \right]. \qquad (16)$$

In order to express also the damper current i_d by the e.m.f. E, we introduce the rotor leakage flux Φ_r of equation (5) in equation (4) and obtain

$$N_d \frac{d\Phi}{dt} + r_d i_d = N_d \frac{x_r}{\omega N_a} \frac{d}{dt}\left(I - \frac{n}{m} i_d \right). \qquad (17)$$

Substituting equation (15) on the right-hand side of equation (17) and introducing E rather than Φ on the left-hand side by use of equation (7), gives

$$N_d \frac{\Phi_0}{E_0} \frac{dE}{dt} + r_d i_d = N_d \frac{x_r}{\omega N_a}\left[\frac{dE/dt}{X + x_s + x_r} - \frac{n}{m} \frac{X + x_s}{X + x_s + x_r} \frac{di_d}{dt} \right]. \qquad (18)$$

For the first term on the right-hand side of this relation we can express the fraction before the bracket by equation (6), giving the ratio of rotor leakage flux Φ_{r0} to stator current I_0 under steady-state condition. On the other hand, the sum of all the reactances in the first denominator within the bracket is given by the quotient of e.m.f. E and current I_0 for the steady state. There results, therefore, neglecting the differ-

ence of E and E_0, which is insignificant here,

$$\frac{x_r}{\omega N_a}\frac{dE/dt}{X + x_s + x_r} = \frac{\Phi_{r0}}{I_0}\cdot\frac{I_0}{E_0}\cdot\frac{dE}{dt}. \tag{19}$$

However, we can combine this term with the first term on the left-hand side of equation (18), obtaining

$$N_d\frac{\Phi_0 - \Phi_{r0}}{E_0}\frac{dE}{dt} = N_d\frac{\Phi_a}{E_0}\frac{dE}{dt}, \tag{20}$$

where Φ_a is the armature flux as difference of pole and rotor leakage flux in the steady state. The last term on the right-hand side of equation (18) can be written

$$\frac{x_r}{\omega}\left(\frac{N_d}{N_a}\right)^2\frac{X + x_s}{X + x_s + x_r}\frac{di_d}{dt} = L_r\frac{di_d}{dt}, \tag{21}$$

so that the entire equation (18) for the damper current may be expressed as

$$N_d\frac{\Phi_a}{E_0}\frac{dE}{dt} + r_d i_d + L_r\frac{di_d}{dt} = 0. \tag{22}$$

We see therefore that L_r is the self-inductance of the damper winding with the rotor leakage flux. It is expressed in equation (21) by the rotor leakage inductance x_r/ω viewed from the armature, reduced by the square of the turn ratio of damper and armature circuits, and multiplied by a diminution factor

$$\rho = \frac{X + x_s}{X + x_s + x_r}, \tag{23}$$

which expresses the effect of the armature reaction on the damper circuit.

Our problem now consists in solving simultaneously the differential equations (16) and (22), both containing as dependent variables E and i_d only, but with a non-linear term $i_s(E)$ constituting the magnetic characteristic of the machine. In order to solve these equations by a simple graphical method, we note that in the main equation (16) the damper current i_d plays merely the part of a correction term for the

variation of the main flux and e.m.f. with time. It may be sufficient therefore if we satisfy equation (22) by a good approximation for the general variation of i_d and introduce that damper current into the main equation (16). Thus the assumption may be made, derived from the form of equation (22), that the damper current changes nearly exponentially with time, so that

$$\frac{di_d}{dt} = -\frac{i_d}{T}, \tag{24}$$

still allowing for a slow variation of the time constant T, the magnitude of which is left undetermined.

By introducing equation (24) in equation (22) and denoting the time constant of the damper with the rotor leakage flux

$$T_r = \frac{L_r}{r_d}, \tag{25}$$

we obtain, with use of equation (9),

$$\frac{N_d}{N}\frac{\Phi_a}{\Phi_0} T_m \frac{dE}{dt} + r_d \left(1 - \frac{T_r}{T}\right) i_d = 0, \tag{26}$$

by which i_d is determined as a function of dE/dt. Inserting the damper current i_d in the last term of equation (16), this term becomes

$$r\rho n i_d = \rho \frac{N_d^2 \Phi_a}{r_d} \cdot \frac{r}{N^2 \Phi_0}, \frac{T_m dE/dt}{1 - T_r/T} = \frac{T_d}{T_p} \cdot \frac{T_m dE/dt}{1 - T_r/T}. \tag{27}$$

The central expression is so written that we see immediately that the first two fractions give the ratio of two time constants. These are: a damper time constant T_d, determined by resistance and number of turns of the damper circuit and by the armature flux linked with this circuit and therefore giving a value different from T_r in equation (25), and the main pole time constant T_p, determined by the corresponding constants of the excitation winding and already given by equation (10). By combining the last term of equation (16), given by equation (27), with the left-hand side of equation

(16), we now obtain

$$T_m\left[1 + \frac{T_d/T_p}{1 - T_r/T}\right]\frac{dE}{dt} = e(t) - r\left[i_\mu(E) + \frac{mE}{X + x_a + x_r}\right]. \quad (28)$$

Herein the right-hand side contains the quotient of e.m.f. and total reactance of the circuit constituting the steady state network characteristic, as shown in Fig. 3. If we now introduce on the left-hand side the rotor pole time constant T_p of equation (10), rather than the machine time constant T_m of equation (9), we obtain the differential equation for the e.m.f. of a saturated machine with damper circuits, in the final form

$$\left[T_p + \frac{T_d}{1 - T_r/T}\right]\frac{d(E/E_0)}{dt} = \frac{e(t)}{e_0} - \frac{\Sigma i}{i_0}. \quad (29)$$

The last term Σi on the right-hand side denotes the sum of the internal and external steady-state magnetizing current under the e.m.f. E, this sum being always a given function of E, see Fig. 3.

The expression in the bracket on the left-hand side of equation (29) is a resultant time constant according to which the e.m.f. E, and simultaneously the damper current i_d, can vary. Hence we have the conditional equation

$$T = T_p + \frac{T_d}{1 - T_r/T}, \quad (30)$$

which can be written as a quadratic equation

$$T^2 - T(T_p + T_d + T_r) + T_pT_r = 0. \quad (31)$$

Thus two possible different time constants appear, corresponding to the two rotor circuits.

With moderate rotor leakage, as in actual machines, the rotor leakage time constant T_r is small compared with the other time constants. Then the two solutions of equation (31) are in very good approximation

$$T_1 = T_p + T_d; \qquad T_2 = \frac{T_r}{1 + \frac{T_d + T_r}{T_p}} \cong T_r, \quad (32)$$

where the last term on the right-hand side is a further approximation for relatively small T_d. The large main flux time constant T_1 is given by the sum of the time constants of field winding and damper circuit, both related to the main fluxes in armature and poles. The small leakage flux time constant T_2, on the other hand, is given chiefly by the rotor leakage flux from pole to pole closing itself through the damper circuit.

4. GRAPHICAL REPRESENTATION OF THE SOLUTION.

We see now that our differential equation (29) has two solutions, a forced solution belonging to the main flux in the pole core, and a free solution belonging to the rotor leakage flux between the poles. The forced solution is given by

$$T_1 \frac{d(E/E_0)}{dt} = \frac{e(t)}{e_0} - \frac{i_\mu(E) + mI_1}{i_0} = \frac{\Delta i_1}{i_0} \qquad (33)$$

and determines by the large value of T_1 a slow transient

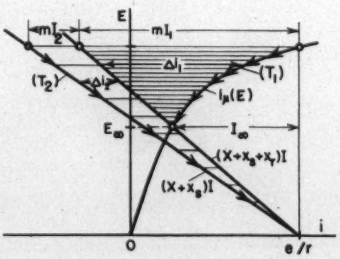


FIG. 4. Graphical representation of the slow and the rapid transient changes of state.

change with time of the e.m.f. E and of the corresponding main pole flux. This flux is excited by an ohmic current e/r determined by the excitation voltage $e(t)$, from which the magnetization current i_μ for the main circuit and the steady state armature reaction current mI, both corresponding to E, are to be deducted. For constant excitation voltage e, the right-hand side of Fig. 4 gives a graphical representation of

the change of e.m.f. E with time which is for every E proportional to the difference current Δi_1, closely cross-hatched in Fig. 4.

The free solution is given by the remaining differential equation

$$T_2 \frac{d(E/E_0)}{dt} = -\frac{mI_2}{i_0} = \frac{\Delta i_2}{i_0} \tag{34}$$

and determines by the small value of T_2 a rapid transient change with time of the rotor leakage flux. This flux is excited by the damper current but is not influenced either by the excitation voltage or by the iron magnetizing current i_μ.

The change of this leakage flux and its currents is represented graphically by the left-hand side of Fig. 4, which is widely cross-hatched and shows as exciting current only that of the rotor leakage reactance x_r. With no saturation in the rotor leakage circuit, the leakage flux and the corresponding part of the damper current decay exponentially, since Δi_2 is then proportional to E. With saturation of the leakage flux we can apply equation (34) graphically in the same way as described for equation (33).

The total stator current is always given as the sum of the forced and free solutions, i.e., $I_1 + I_2$, as already indicated in equation (14).

While in Fig. 4 the forced Δi_1 decreases towards zero at the intersection of the saturation and the network characteristics, yielding a steady state voltage E_∞ and a sustained current I_∞ in the stator, the free Δi_2 decays towards zero at the zero line, leaving no current or voltage in the damper winding. The rate of decrease is slow for the forced current Δi_1 with the large time constant T_1 but rapid for the free current Δi_2 with the small time constant T_2. Hence Δi_2 will have nearly vanished when Δi_1 begins to move.

In Fig. 5 the development of the short-circuit current with time is shown when the terminals of a saturated generator are suddenly short circuited and the excitation voltage is kept constant, the generator previously having been without load. The magnetic no-load characteristic of the machine, $i_\mu(E)$, in conjunction with the stator leakage characteristic $x_s I$ and the

total leakage characteristic $(x_s + x_r)I$ determines completely the transient state, including the initial conditions.

The current of unbalance Δi_1 as defined by equations (28) and (33) is given by the horizontal width of the closely cross-hatched area of Fig. 5*a* as the difference between the previous excitation current $i_1 = e/r$ and the sum of the no-load magnetizing current $i_\mu(E)$ and the steady-state short-circuit current $I_1 = E/(x_s + x_r)$, the latter being plotted in the rotor scale backwards from i_1 as a straight line. Since Δi_1 is dependent only on E, the curve $E(t)$ can be drawn as in Fig. 5*b*, either in step-by-step construction with the always known value of dE/dt or by a quadrature yielding t as a function of E

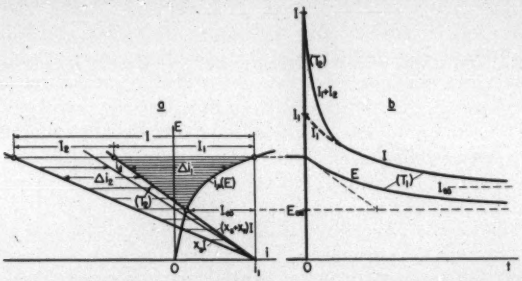


FIG. 5. Development of slow and rapid transient currents at sudden short circuit of the terminals: (*a*) Diagram of characteristics; (*b*) Change with time.

after separation of the variables in equation (33). To every E the current I_1 can be correlated by means of the total leakage characteristic. This slow transient stator short-circuit current corresponds to the slow variation of the main pole flux with the saturated time constant T_1.

In addition to this, a rapidly varying flux is produced by the current Δi_2 shown as widely cross-hatched between the two leakage characteristics in Fig. 5*a* and acting on the rotor leakage path only. This damper current is balanced by an additional stator current I_2, both decaying rapidly with an exponential time constant T_2 according to equation (34). Instead of a step-by-step decrease of Δi_2 down to zero, or of a quadrature of equation (34), we may turn the stator leakage

characteristic x_s towards the complete leakage characteristic $x_s + x_r$ with a velocity corresponding to the time constant T_2. Then any intermediate position of this line always determines the left-hand end up to which the total stator current $I_1 + I_2$ has to be measured. Fig. 5*b* shows the rapidly decaying additional current I_2 which forms a large initial increase of the short-circuit current.

While the rapid transient current changes exponentially with time, so long as no saturation prevails in the leakage paths, the slow transient current deviates widely from an exponential variation due to the effect of saturation in the main magnetic circuit.

In Fig. 6 the sudden interruption of a terminal short circuit of a saturated generator is shown, the excitation voltage

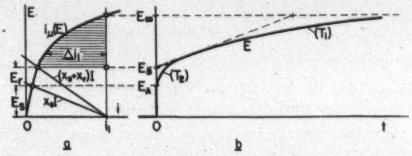


FIG. 6. Development of slow and rapid transient recovery voltages at sudden interruption of a short circuit at the terminals: (*a*) Diagram of characteristics; (*b*) Change with time.

again being kept constant. Here, according to Fig. 6*a*, both leakage characteristics determine the previous steady-state condition of the fluxes by intersection with the no-load characteristic. The characteristic x_s gives the stator main flux, the characteristic $x_s + x_r$ gives the rotor pole flux. At the instant of interruption of the short circuit, the rotor pole flux is kept constant by the field winding but the stator flux is also kept constant by the additional action of the damper winding. Thus the e.m.f., corresponding to the rotor pole flux, starts with the value E_B and increases slowly according to the current of unbalance Δi_1 which lies between the characteristic $i_\mu(E)$ and the vertical line through i_1, the latter constituting the open terminal characteristic.

The stator voltage, on the other hand, jumps at the instant of interruption from zero to the value E_A, equal to the

previous stator leakage voltage, and increases rapidly towards the higher value of the e.m.f. E, as in Fig. 6*b*, due to the decaying rotor leakage flux. Thus with interruption of the circuit the voltage at the terminals of the generator increases instantaneously due to the stator leakage; it then increases rapidly due to the rotor leakage, and it further increases slowly due to the main flux in the machine, finally attaining the no-load voltage.

The rapid transient voltage is nearly exponential with the time constant T_2; the slow transient voltage is distorted by saturation and is to be derived by the shaded area in Fig. 6*a* with the time constant T_1. It may be noted that the scales of the characteristics and times in Figs. 5 and 6 are chosen the same so that the rates of change of the voltages are directly comparable.

5. MAGNETIC TIME CONSTANTS.

The development of many of the transient conditions which may occur in the operation of saturated synchronous machines has been shown in a previous paper,[1] in which, in analogy to non-saturated circuits, there was assumed a well-known quadratic equation for the time constants. In the present paper a more detailed analysis of the magnetic linkages has led us to a similar quadratic equation (31), except that we have now arrived at relations determining numerically all the time constants, and the damper time constants appear here separated into the magnitudes T_d, belonging to the main armature flux, and T_r, belonging to the rotor leakage flux. Furthermore, we have shown here by equation (23) to what extent the damper time constants are influenced by armature reaction.

The damper time constant of the main flux is, according to equation (27),

$$T_d = \rho T_p \left(\frac{N_d}{N}\right)^2 \frac{r}{r_d} \frac{\Phi_a}{\Phi_0} = \rho \frac{N_d \Phi_a}{r_d i_d}. \tag{35}$$

The last expression on the right-hand side is obtained by substituting equation (10) for T_p and introducing as the exciting transient the damper current rather than the field current, these two being in the ratio of their turn numbers.

For numerical computations the central expression of equation (35) is convenient also. It shows that this damper time constant T_d is only a fraction of the main flux time constant T_p, since the damper cross section, determined by N_d^2/r_d, is always much smaller than the field coil cross section, determined by N^2/r. Furthermore, the damper time constant is reduced by the factor ρ representing by equation (23) the influence of armature reaction.

Similarly, we obtain for the damper time constant of the rotor leakage flux, according to equations (25) and (21), and using equations (6) and (10),

$$T_r = \rho T_p \left(\frac{N_d}{N}\right)^2 \frac{r}{r_d} \frac{\Phi_{r0}}{\Phi_0} = \rho \frac{N_d \Phi_{r0}}{r_d i_d}, \qquad (36)$$

which again is a small fraction of T_d, in view of the additional effect of the ratio of the rotor leakage flux Φ_{r0} to the main pole flux Φ_0.

By equations (10), (35), (36), we can compute readily the resultant time constants of equation (32) for any given machine. We have to note, however, that due to the factor ρ both damper time constants are dependent on the load, which in equation (23) is expressed in terms of the external reactance X. At no load we have $X = \infty$ and therefore $\rho = 1$, indicating no armature reaction and full value of the damper time constants. At terminal short circuit, we have $X = 0$, and thus, with equal rotor and stator reactances, for example, $\rho = \frac{1}{2}$. This shows that the additional armature currents, developed simultaneously with the damper currents, reduce the value of both damper time constants with increasing load towards about one-half at short circuit. However, at any actual load this effect is slight, since X then outweighs the leakage reactances in equation (23).

6. VARIOUS DAMPER CIRCUITS.

Actual damper constructions of synchronous machines differ to some extent from our simplifying assumptions. In particular, the rotor leakage field is not the only flux linked with the damper circuit. Fig. 7 shows that a further flux Φ_l may surround the damper conductors, closing itself partially through the damper slots, partially around the end rings of

the damper. This additional flux is produced by the damper currents i_d only, but not by the stator currents I. If we include this additional leakage flux in the fundamental conditions, equations (1), (2), (3), (5) remain unchanged. In equation (4), however, we have to add on the right-hand side a small term $L_\delta di_d/dt$, giving the induced voltage by the independent damper leakage inductance L_δ. Correspondingly we have to add to the time constant T_r, related to the rotor leakage flux by equation (36), a small time constant T_δ due to this inherent damper leakage flux. This magnitude is not influenced by armature reaction and therefore does not contain the factor ρ. The time constant T_d in equation (35) remains

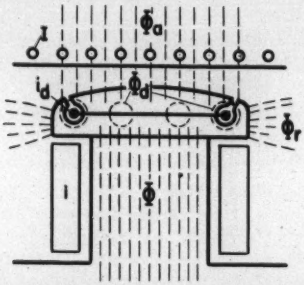


FIG. 7. The magnetic fluxes in space linked with a damper winding in the pole shoes of salient poles.

unchanged since it is determined by the main armature flux only.

Actual dampers, as shown in Fig. 8, are built for the most part in the form of squirrel cages, with spatially distributed damper bars. Such circuits are very closely linked with the distributed armature currents but much less closely with the lumped field winding. Hereby, a further leakage between damper and field winding occurs, increasing the independent time constant T_δ. On the other hand, the distributed currents of squirrel-cage dampers are not completely linked with the main flux. Hence the damper time constant T_d becomes somewhat smaller than in equation (35). This can easily be

taken into account by including in the number of turns N_d of the damper an additional linkage factor according to familiar rules.

Since the actual distribution of the damper currents over the parallel damper bars of a squirrel cage is free, the entire damper flux contains in space fundamental as well as harmonic fields. Therefore, there may appear here a whole series of time constants rather than the two derived for lumped circuits. However, the amplitudes of the harmonic fields, voltages and currents will be small, since they are determined merely by difference effects moderate in magnitude.

Even if there is no actual damper winding, but the pole cores or pole shoes are made of solid steel, as in Fig. 9, eddy currents can develop in such solid parts acting qualitatively

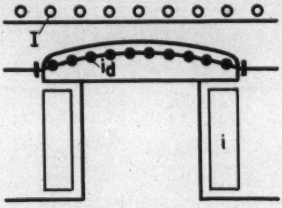


FIG. 8. Squirrel-cage damper winding in contrast to the lumped circuit of Fig. 7.

as damper currents. Quantitatively their time constants T_d and T_r are sometimes smaller because of the high electric resistance of the steel. They can be computed [2] readily for any cross section of the steel poles and may contribute particularly to T_d. In Fig. 9 the spatial distribution of these damping eddy currents is shown in comparison to the distribution of the field currents. Since the distributions differ from each other, residual leakage is caused and here also a small independent time constant T_i is developed in addition to T_r. A solid rotor yoke has the same effect in principle; however, the linkage with both stator and rotor windings is very small and therefore the independent leakage is large, while the contribution to T_r vanishes.

[2] R. Rüdenberg, "Elektrische Schaltvorgänge" (a book), 3rd edition, Berlin, 1933, p. 70.

If a copper damper and solid steel parts are acting together, as in many actual machines, the effects of both merely add, increasing all the main and leakage time constants.

In cylindrical rotors for turbo-generators, as shown unrolled in Fig. 10*a*, the linkage between field currents and eddy currents in the solid steel parts is relatively large. In Fig. 10*b* the distribution of the field currents and of the damping eddy currents along the circumferential direction is shown for both the currents in the solid iron and in the better conducting

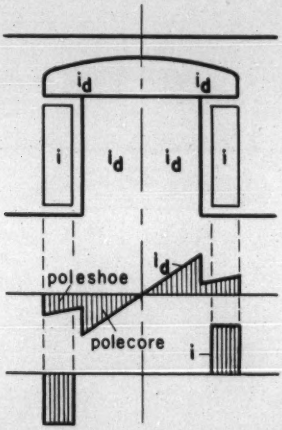


FIG. 9. Distribution of damping eddy currents in the solid steel parts of salient poles.

metal wedges closing the slots. We see that the distribution of the entire damper currents i_d is similar to that of the field currents i. Hence their linkage with the main flux is nearly perfect and a relatively large main flux damper time constant T_d is developed by the large steel cross section, according to equation (35), while the additional leakage and therefore T_s are only slight. Since the currents in the metal wedges are completely linked with the armature flux, as is the case with an ideal damper, the quotient Φ_a/Φ_0 remains in equation (35) for this part of the damper circuit. However, the eddy

currents in the solid teeth and particularly in the core are linked with the total pole flux Φ and therefore we may cancel the quotient Φ_a/Φ_0 in equation (35) for their part. All three parts of the currents, in wedges, teeth and core, are subjected to armature reaction by the factor ρ.

Figure 10*c* shows the distribution of field current and eddy currents along the radial direction of the solid rotor. The large currents in the metallic wedges flow above the field currents, surrounding the armature flux only and leaving the rotor leakage flux between field windings and wedges below. This corresponds exactly to our assumption in Fig. 2 and

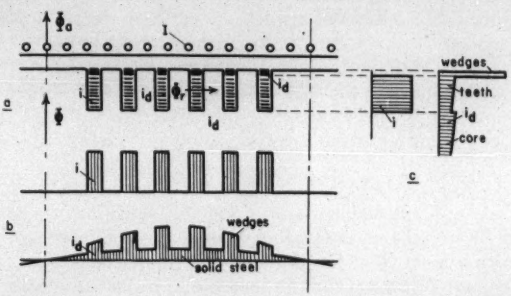


FIG. 10. Distribution of damping eddy currents in the solid metal parts of cylindrical rotors: (*a*) Unrolled rotor surface with winding, wedges and teeth; (*b*) Circumferential distribution of exciting and eddy currents; (*c*) Radial distribution of exciting and eddy currents.

therefore the wedge currents give a leakage time constant T_r as in equation (36). The eddy currents in teeth and core, however, differ in their radial distribution from that of the field currents and surround the entire pole flux. Hence they give, besides their participation in T_d, an additional time constant T_δ with no armature reaction factor, due to the inherent leakage of the solid steel damper circuits.

Although the spatial development of the eddy currents in the solid cross section of the rotor is quite free, Fig. 10 indicates that the distribution follows a line which may be between the nearly sinusoidal distribution of the armature currents and

the more trapezoidal distribution of the field currents. Hence the difference from both of them is fairly small and thus only very slight higher harmonics of the damper currents are developed, and the corresponding additional time constants will be insignificant.

As to the numerical values of the damper time constants for machines of the usual design, the copper cross sections of damper cage winding and field winding, determinative for the middle expression in equation (35), are mostly in the ratio 1/5–1/20. Thus the main flux damper time constant T_d may be of the order of 1/10 of T_p, which means about 0.3–1.2 sec. By eddy current formation in the solid steel circuits this value may be doubled or tripled.

The rotor leakage flux, on the other hand, is mostly in the ratio 1/5–1/10 of the main flux. Its time constant T_r therefore, with regard to the last fraction in the middle expression of equation (36), is of the order of 0.1 sec., including a slight increase by eddy-current effects.

7. RAPID TRANSIENT EFFECTS.

The fundamental equations (1) and (13) for the stator circuit represent merely the amplitudes of voltages and currents which vary harmonically with frequency ω. It is well known, however, that at switching processes in the stator circuit, superposed transients with different variation with time may appear which prevent any discontinuity in the momentary values of the stator voltages and currents. Such additional currents flow without impressed voltage and suffer a rapid decay. They are determined in their initial values by the instantaneous variations of the amplitudes in our previous time diagrams. With sudden inductive loading or short-circuiting of the terminals, there appear direct currents in the stator circuit, decaying with the time constant of this circuit. With capacitive loading or sudden interruption of the circuit, there appear damped oscillations, the natural frequency of which is given by self-inductance and capacitance of machine and load.

A further question in the same direction may be considered, namely, whether the stator equations (1) and (13), containing the amplitudes of sinusoidal voltages, remain sufficiently

accurate although our solutions yield exponential or saturated variations of these amplitudes with time.

In order to prove the admissibility of equation (1), we go back to the differential equation of the stator circuit

$$e = Ri + L\frac{di}{dt}, \tag{37}$$

in which L denotes a constant total self-inductance and R the resistance of the stator circuit. The current i may follow a damped sinusoidal variation

$$i = I\epsilon^{-\frac{t}{\tau}} \cdot \epsilon^{j\omega t}, \tag{38}$$

containing an exponentially varying amplitude, the time constant τ of which is kept arbitrary in order to approximate the form of our previous solutions. Inserting equation (38) in (37) gives the amplitude equation

$$E = \left[\left(R - \frac{L}{\tau}\right) + j\omega L\right] I. \tag{39}$$

Hence the inductive component of the impedance is not altered for any value of τ, only the ohmic component is decreased by the action of a decaying current, the decrease being greater the smaller the time constant τ. So long, however, as

$$\tau \gtrsim \frac{1}{\omega} = \frac{1}{377} \text{ sec.} = 2.65 \cdot 10^{-3} \text{ sec.}, \tag{40}$$

the additional ohmic term L/τ is negligibly small compared with the inductive term of equation (39), as is also the true resistance term R in all machines of considerable size. Since the smallest actual time constant of the rapid transient variation is of the order of 0.1 sec., i.e., nearly 40 times the numerical value in equation (40), we see that equation (1), containing inductive components only, is sufficiently accurate for the varying amplitudes as well.

It is well known that the experimental determination of the leakage of synchronous machines is a rather intricate problem and that the Potier method, and other steady state measurements as well, do not secure accurate results. This

is due partly to saturation effects and partly to the unknown subdivision into stator and rotor leakage. In the light of the present investigation the sudden interruption of a terminal short circuit presents itself as a useful method for the determination of both stator and rotor leakages.

For usual machines, the active periods of the three voltage ascents in Fig. 6*b* are very different from one another, namely: zero, about a tenth of a second, and several seconds. In consequence, not only can the voltages E_A and E_B be measured with accuracy from an oscillogram but both parts can be separated easily, giving directly the rated stator leakage and rotor leakage voltages if the machine was previously excited for a short-circuit current equal to rated current. With other prior currents the effect of saturation on the paths of both leakage fluxes can be measured separately.

This complete test can be made, of course, only with machines having material damper effects, since otherwise the point E_A disappears in the oscillogram and merely the total leakage E_B can be determined. The large time of ascent of the slow transient voltage, present with open-circuited terminals of the machine, favors decisively the accurate determination of point E_B in contrast to any other test with short-circuited terminals which gives a much smaller duration of the slow transient state. Since the machine with rated short-circuit current operates under nearly unsaturated condition of the main flux, the slow transient ascent is exponential in the beginning and this fact may be used for an even more exact determination of E_B.

The rapid transient phenomena produced by the rotor leakage damper time constant T_r have significance only with sudden switching processes in the stator, such as making or breaking a circuit, occurring in times which are small compared with the value of T_r. All the many other transient procedures in the operation of synchronous machines, however, such as voltage control by a field regulator, or mechanical oscillations of the rotor during disturbances of stability, have periods of fluctuation of the order of seconds. Since this time is very large compared with T_r but not with T_p, such procedures initiate only slow transient phenomena of the main flux, but never produce rapid transient disturbances of

balance of the leakage flux. Rather the rotor leakage flux follows all slower fluctuations without any delay just as the stator leakage flux does, both acting in complete unison, so that no separation of their effects is necessary.

8. SUMMARY.

The sudden development of short-circuit currents, and of recovery voltages at interruption of currents, depends on the effects of stator and rotor leakages as well as of saturation in the generating synchronous machines. The non-linear problem of the interaction of these features with the performance of the main windings and auxiliary damper circuits is solved analytically.

This leads to a simple graphical representation which is numerically determined by the well-known magnetic and electric characteristics of machine and network and by two time constants of very different magnitudes which are derived from the ordinary data of the machines. Hence the variation with time, of short-circuit currents and of recovery voltages, can easily and rigorously be computed for every given example.

The deviations of actual damper circuits from the ideal form and the effects of eddy currents in solid steel parts of the rotor are discussed so far that numerical results can be derived readily.

A new transient test for synchronous machines is suggested by which rotor and stator leakage voltages can be measured separately, avoiding the uncertainties of the usual steady state leakage tests.

Measuring the Elastic Airplane.—WELCOME W. BENDER, JR. (*Instruments*, Vol. 15, No. 4.) With the development in recent years of large high performance airplanes, the structural engineer has had to give increased attention to the problems of elastic deflections of the airplane, rather than to designs which are investigated for strength alone. He has now become well aware that the physics of structures which are considered elastic, rather than rigid bodies, encompasses the broad field of vibrations and a dozen related specialized lines of study, which are all in one way or another connected with the dynamic performance of the airplane. When these engineering problems began to be realized in their true perspective a few years ago, effective engineering was hindered by the lack of actual data on the many related phenomena, nor were reliable test instruments available to the researcher who would investigate the problems. Notable results have recently been released by the Glenn L. Martin Company whose experience in vibration research dates back to its design of large transoceanic flying boats several years ago. While the Martin engineers were primarily interested in vibrations in their initial researches, they have continually enlarged their experience and improved their technique so that they are now able with a single set of equipment to measure accurately vibrations, accelerations and strains, whether they be steady or oscillating, recurring or transient. Measurements have been made many times, not only on all sorts of routine ground vibration tests, but also on aircraft in flight, takeoffs, landings, maneuvers, and even during minor accidents. The Martin equipment, since it is electrical, eliminates the multitude of dynamic and inertia errors to which purely mechanical systems in themselves are subject whenever an attempt is made to adapt them for so wide a range of service conditions as are experienced in aircraft use. It became apparent at an early date that an accelerometer type pickup that could be readily mounted on any part of the airplane must be designed first, and the appropriate amplifying and recording equipment developed around it. The design which is now being made available to the industry is approximately $\frac{1}{8}''$ in diameter by $\frac{1}{4}''$ high and weighs less than two ounces. It can be used to measure accelerations or vibrations from zero frequency up to 100 cycles per second and is linear in its response up to twenty times the acceleration of gravity. The amplifiers are designed to operate in conjunction with a standard commercial recording oscillograph which photographs the amplified pickup response on permanent roll paper film records from 6″ to 12″ wide.

R. H. O.

AN ELASTIC THEORY FOR RUBBER.

BY

ELMER LATSHAW.[1]

The elastic properties of vulcanized rubber present a difficult problem to generalize, due mainly to the large number of compounds in common use. There are good reasons for many kinds and grades and a characteristic common to all is large elastic strain. Even in the best grades of rubber these strains are not perfectly elastic, but this is also true for most all other materials including spring steels.

An elastic strain theory will be derived for a specific group of rubber compounds, namely the spring stocks. These are dense, strong rubbers, highly elastic within the working stress range and capable of supporting loads under continuous vibration and shocks. To be of real value, the strain law should closely predict strain for the range of working stress, or more generally for the elastic range, and indicate the trend at very high stresses where inelasticity becomes more prominent.

Figure 1 shows a cylindrical element of rubber having an area A_0 and length h when unstressed. If a uniform stress is imposed perpendicular to the end area, the length changes to H and the area to A, these symbols applying whether the stress is tension or compression. The usual definition for strain, change in length due to stress divided by the unstressed length, is comprehensive and descriptive for the large strains in rubber.

$$\text{Strain} \qquad S = (H - h)/h. \tag{1}$$

$$\text{Strain Ratio} \qquad R = H/h = (S + 1). \tag{2}$$

Denote the unit stress by p and modulus of elasticity by E for both tension and compression. By assuming Hooke's

[1] "Railway Truck Engineer," The, J. G. Brill Company, Philadelphia, Pa.

law of elasticity applicable to rubber yields the two equations:

Stress $$p = ES. \quad (3)$$

Load $$P = EA_0S. \quad (4)$$

Equations (3) and (4) are quite accurate for metals and hold good up to the elastic limit, but such strains are very small approximating ($S = .005$) for spring steel, and within such limits there is no perceptable change in the area A_0. It is easy to show by tests that these equations predict small strains in rubber and equally easy to show that they fail to

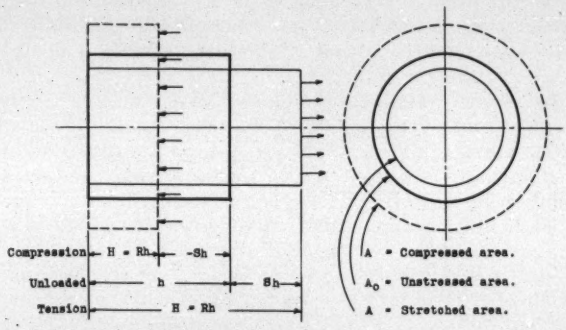


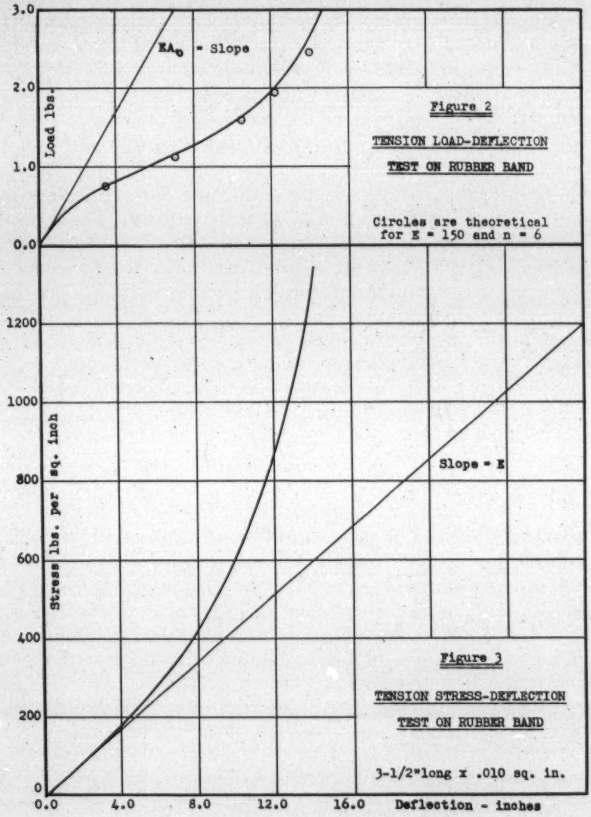
Figure 1

Shape alterations of a rubber cylinder resulting from pure tension and compression stresses.

indicate large strains accurately. The curves in Fig. 2 and Fig. 3 were plotted from an actual tension test on one compound and are typical of other spring stocks as will be seen later. Since the tests depart widely from the constant slope modulus lines of linear equations (3) and (4), it is evident that Hooke's law is only valid for very small strains in rubber, say up to ($S = .25$) and altogether inadequate for high values.

A first consideration in the revision of Hooke's law of elasticity is the high bulk modulus of rubber. Compounds classed as spring stocks [2] are very dense or solid. Tests show

[2] Cellular or sponge rubbers are excluded. Spring stocks are vulcanized under high pressure and have a specific gravity approximating unity.



the bulk modulus [3] to vary from $K = 300{,}000$ lbs. per sq. in. for soft stocks to $K = 400{,}000$ for the hard grades. These are large values compared to the direct modulus $E = 150$ to 400 lbs. per sq. in. for the same range of hardness. Hence,

[3] Bulk modulus values are results of tests made by the United States Rubber Company.

no appreciable error will be introduced by assuming the volume to remain constant for the full range of elastic strain. This assumption leads to the equation:

$$\text{Volume} \qquad V = A_0 h = AH = ARh. \tag{5}$$

Equations (3) and (4) are linear due to the assumption that E is constant and this is quite accurate for metals. In order to formulate large elastic strain for rubber, E might be assumed a variable function of stress or strain. This course is not taken due to the departure from the common idea of modulus being a fixed entity for any substance. To adopt a constant direct modulus for rubber requires that it be restricted to some definite load status. A specific definition for E is the slope of the stress strain curve for zero strain or load. Thus, it is associated equally with tension and compression and conforms to the definition for metals where strain is small or nearly zero. Hence:

Modulus of Elasticity

$$E = \frac{dp}{dS}, \qquad \text{where} \qquad S = 0. \tag{6}$$

With E defined as a constant, it is necessary to modify equation (3) in order to yield a stress-strain graph similar in shape to the test plotted in Fig. 3. This might be done by substituting the unknown function $F(S)$ for S where $F(S)$ is to be derived by a consideration of the physical action of the rubber when subjected to large strains. By the use of $F(S)$, constant volume principle (5) and modulus of elasticity (6) equations (3) and (4) can be rewritten in the following form which is applicable to rubber:

$$p = EF(S), \tag{7}$$

$$P = EA_0 \frac{F(S)}{R}. \tag{8}$$

Now if all grades of spring rubbers presented similar stress-strain graphs in relation to the E lines, $F(S)$ might be derived empirically. However, as will be shown later, all grades do not follow the same $F(S)$ function since the curves are not asymptotic to the same strain ordinate. Determining a

strain function empirically for each grade would suggest a difference in the strain behavior of each. It is reasonable to expect similar materials to follow the same basic strain law in which case the law must contain constants which depend on the differences in the various grades.

It is helpful to consider vulcanized rubber made up of very small unit cells sensibly equal in composition. Each cell will contain hydrocarbon atoms, some of which are combined with the sulphur present, also an average amount of the fillers. These cells are apparently quite rigid and resist shape distortion from stress, since the X-ray [4] shows stretched rubber

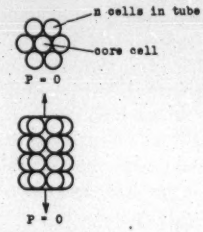


Figure 4a

Typical cell arrangement in unstressed rubber.

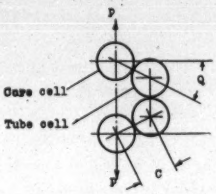


Figure 4b

Cell movement due to tensile stress.

to be made up of crystalline bodies arranged in a fiber pattern, and when the stress is relieved the crystalline bodies are still present but orientated at random. Hence, it appears that the shape alteration of the mass under large strains calls for a redisposition of the unit cells and not a shape change in the cells geometrically similar to the entire mass.

For the purpose of illustrating strain by cell motion, a tensile force is imposed on the axis of a cylindrical element, Fig. 4*a*, consisting of a large number of cells. Along the center is a continuous line of cells. Surrounding the core is a tubular volume also filled with cells. All cells are shown spherical for convenience and no attempt will be made to postulate their real shape. While it is probable that the cells

[4] "X-ray Analysis of Rubber," by Dr. H. J. McNicholas, National Bureau of Standards.

are disposed at random originally, this regular pattern is assumed as an average for further analysis.

The entire cross sectional area of the rubber mass, Fig. 1, consists of cylindrical groups of cells like Fig. 4*a*, the lengths extending through the mass for a distance h when unstressed. If all of these cylindrical groups are stretched by an axial load and the cells maintain their shape, then elongation without rupture calls for drawing the tube cells into the core line, filling up the space which would otherwise occur between the original core cells due to elongation.

Assuming all cells of like size and shape results in the ideal pattern shown in Fig. 4*a* with six tube cells for each one in the core, this leads directly to the limit of 600 per cent. elongation for possible elastic action if the cell distribution is the perfect arrangement shown. If rupture does not occur at 600 per cent. stretch, then the cylindrical groups must start robbing from the adjacent elements which obviously involves inelasticity since there would be no identification remaining as to the source of the various cells and their complete return prevented when unloaded. If the cell arrangement is not perfectly regular like Fig. 4*a*, it is possible that adjacent cylindrical elements may both claim certain cells leading to inelasticity below 600 per cent. elongation.

It can easily be shown that five tube cells for one core cell, all the same size and spherical in shape, is a possible arrangement provided the two types are staggered along the axis. Allowing some oddities in shape, eight tube to one core cell appears to be the maximum ratio since the tube area is eight times the core area.

Now, let's generalize the cell structure, letting n equal the ratio of tube to core cells. Then, the greatest perfectly elastic elongation will be limited to $100n$ per cent. If the unstressed cylinder Fig. 4*a* has an area A_0 and loaded by a tensile force P then by equation (5) the unit tensile stress will be:

$$p = \frac{PR}{A_0}. \qquad (9)$$

Figure 4*b* shows the cells spaced by C and rotated through an angle Q by the tensile strain. The strain moment will be:

$$M_s = pC \cos Q. \qquad (10)$$

Denoting the resistance to cell rotation by the elastic constant E and noting that rotation is proportional to S then the moment of resistance to rotation will be:

$$M_r = CES; \tag{11}$$

also

$$Q = \frac{\pi S}{2n}. \tag{12}$$

Equating (10), (11) and (12) gives:

$$p = ES \sec \frac{\pi S}{2n} \qquad \text{(Tension)} \tag{13}$$

by (9)

$$P = \frac{A_0 ES}{R} \sec \frac{\pi S}{2n} \qquad \text{(Tension).} \tag{14}$$

Hence

$$F(S) = S \sec \frac{\pi S}{2n} \qquad \text{(Tension).} \tag{15}$$

Transposing (13) and (14) to non-dimensional forms:

$$\frac{p}{E} = F(S) \qquad \text{(Tension)} \tag{16}$$

$$\frac{P}{EA_0} = \frac{F(S)}{R} \qquad \text{(Tension).} \tag{17}$$

A compression stress-strain equation can easily be derived directly from the tension equation (13) by quite simple reasoning. It is apparent that a tension strain ratio R denotes the reverse physical process to a compression strain ratio R' if $(RR' = 1)$. For example, by stretching a length h the new length is (Rh), and starting with an unstressed length (Rh) and compressing it by a ratio R' it is shortened to h.

For tension, the cells pulled into the core per unit length is:

$(Rh - h)/h = (R - 1)$ Corresponding to stress p,

and compression extrudes cells from the core per unit length by:

$(Rh - h)/Rh = (R - 1)/R$ Corresponding to stress p'.

Since the unit stress is proportional to unit cell movement:

$$p' = -pR' \qquad (\text{for } RR' = 1). \tag{18}$$

Or a compression strain ratio R' sets up a compression stress equal to R' times the tensile stress from a tensile strain ratio R. Combining (13) and (18) yields:

$$p' = E(R' - 1) \sec \frac{\pi(R' - 1)}{2nR'}. \tag{19}$$

Since R is eliminated from (19) the condition ($RR' = 1$) no longer applies and the primes can be dropped giving the compression equations which follow:

$$p = ES \sec \frac{\pi S}{2nR} \qquad \text{(Compression)} \tag{20}$$

by (9)
$$P = \frac{A_0 ES}{R} \sec \frac{\pi S}{2nR} \qquad \text{(Compression)} \tag{21}$$

and
$$F'(S) = S \sec \frac{\pi S}{2nR} \qquad \text{(Compression).} \tag{22}$$

Transposing (20) and (21) to non-dimensional form:

$$\frac{p}{E} = F'(S) \qquad \text{(Compression),} \tag{23}$$

$$\frac{P}{EA_0} = \frac{F'(S)}{R} \qquad \text{(Compression).} \tag{24}$$

It is to be noted that the elastic equations (13) and (14) for tension differ slightly in form from the compression equations (20) and (21), this being the result of the strain definition selected. By comparing the non-dimensional forms (16) and (17) for tension and (23) and (24) for compression with the original equations (7) and (8), it is seen that they are alike in form. Since the strain functions $F(S)$ are all secant functions, it is clear that all these strain equations resolve to Hooke's law given by (3) and (4) for infinitesimal strains. This indicates Hooke's law to be a special case of this more general law for large elastic strains.

A number of tests will now be cited to support these theoretical elastic strain laws. Fig. 2 shows a graph of a

tension test made by the author on a band of rubber. For this grade $E = 150$ and $n = 6$ and the circles represent values computed by equation (14). The tension strain is accurately predicted up to 400 per cent. elongation.

The strain law was further verified by a series of standard tension tests [5] made on commercial spring stocks. Data given below describe three of the grades tested.

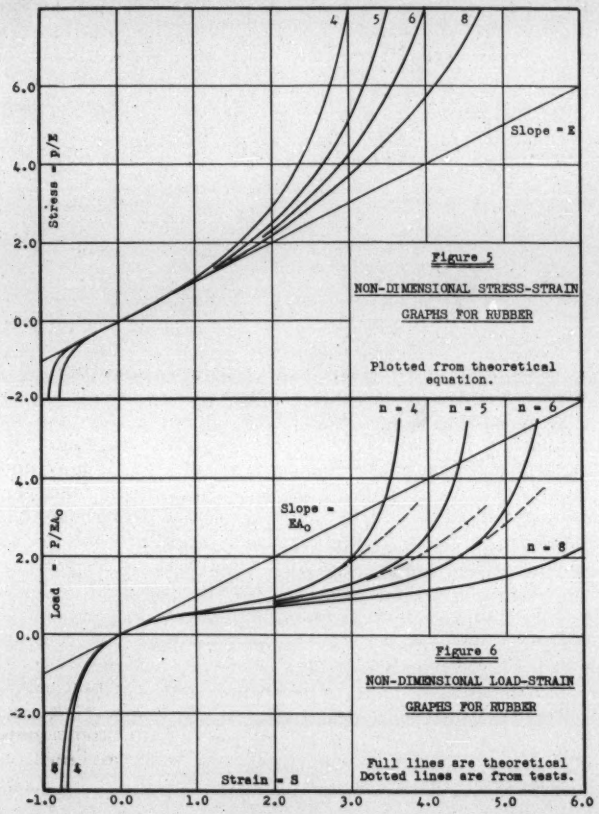
Durometer Hardness.	Direct Modulus E.	Structure n.	Test Agreement.	Elongation at Rupture.
29	165	6	450%	800%
40	225	5	350%	700%
61	420	4	300%	550%

Non-dimensional strain equations (16), (17), (23) and (24) are plotted in Figs. 5 and 6, using four different n values. The tests of the three rubbers given in above table follow these curves accurately up to very high strain values, and the dotted lines branching off from the theoretical curves show where deviation begins. The theoretical curves denote perfect elasticity up to rupture and any departure from elasticity will give more strain than is predicted. It is important to note that the test deviations show the proper trend.

The tension and compression tests made by Henchy [6] were compared with this theory for the compound having $E = 210$ and $n = 5$. All test values agree very satisfactory over the entire tension and compression range. This agreement is of real importance since it supports the idea that E applies equally to both tension and compression strains. Tension strain tests are relatively easy to perform using a band or the standard tension specimen. Much more difficulty is encountered in compression, the cylinder, Fig. 1, must be short relative to the diameter for stability and the ends must be well lubricated to permit end area expansion with negligible friction. Tests described by Henchy [6] and

[5] These tension tests were made by the United States Rubber Company expressly for the purpose of checking this theory. The author is grateful for this important contribution.

[6] "The Elastic Behavior of Vulcanized Rubber," by H. Henchy. Presented at the National Applied Mechanics Meeting, New Haven, Conn., June 22–25, 1932, A. S. M. E.



Kimmich [7] show how pure compression stresses can be accomplished by employing suitable lubricants on the loading faces.

Returning again to the cylindrical cell group shown in

[7] "Rubber in Compression," by E. G. Kimmich. Presented at the meeting of A. S. T. M. Committee D-11, on Rubber Products, Atlantic City, N. J. June 28, 1940.

Fig. 4*a*. A pertinent question is: What determines the identity of the tube and core cells? Their function or identity may depend entirely on chance or controlled by variations in size, shape or adhesive forces between them. Probably under low strains, lines of core cells are selected at random where the most direct alignment occurs, but when the strain becomes large, core cells are forced to operate at uniformly spaced locations so that the action of cell motion will be equalized throughout the volume. Test behavior makes it evident that cell function is not established immediately. The first few test runs show considerable variations and as many as ten cycles or more must be performed on a new specimen before the results will repeat.[8] This was also pointed out by Kimmich [7] in connection with compression tests.

Aside from immediate uses, such as predicting strain for a specific condition of loading, this elastic strain theory may be of some value to the rubber chemist in the study of fillers and compounds. Possibly it will serve to further emphasize the importance of particle size and shape, also the value of uniformity for materials used as fillers. Evidently, the percentage and kind of fillers and other compounding materials influence or establish the structural constant n, since its value changes for different grades. The author was unable to find any commercial compounds corresponding to $(n = 8)$; this may be an interesting structure for the chemist to produce. Experiments with fillers and their effect on the structural constant may support or add to the basic theory; or may suggest another mechanical picture of strain which will prove to be more general or useful. If this work contributes in any way to the development of better compounds or their more efficient use, its purpose will be accomplished.

[8] The tension-strain tests made by the United States Rubber Company to check the strain theory brought out this fact.

Oil-Reservoir Behavior Based Upon Pressure-Production Data.—This is the title of a U. S. Bureau of Mines Report of Investigations (No. 3634) by H. C. MILLER. For many years observations have been made with regard to the most efficient method for the recovery of oil. At best it is a complex problem involving not only a knowledge and application of the fundamentals of fluid flow through porous mediums, but the sources of energy responsible for the movement of oil and gas through reservoir sands and porous rocks to wells must be known to determine the best practices for extracting these hydrocarbons. And with regard to reservoirs undergoing depletion of their contained fluids, pressures and statistics of oil and gas production are fundamental information of utmost importance in a study of the mechanics of production. Such information, covering a long period, now is available on a number of petroleum reservoirs and, if wisely interpreted, should prove of inestimable worth in advancing technical knowledge of the producing characteristics of oil-bearing reservoirs. Mr. Miller undertook the study of information relative to fields with a wide range of producing characteristics. In carrying out the study, customary methods of plotting pressure-production data—for example, reservoir-pressure observations against rates of oil production or against cumulative oil recovery on Cartesian semilogarithmic, and logarithmic charts—seemed to yield no graphic representations of the mechanics of production of the reservoirs which, by themselves, might picture reservoir performance or aid in an interpretation of the reservoir mechanics. It was found, however, that when the cumulative decline of reservoir pressure was plotted on logarithmic charts against cumulative oil (and water) produced the relationship between them is definable by straight lines for large increments of production. From this it became apparent that there was promise not only for representing graphically the producing performance of petroleum reservoirs, but a means for determining the relative efficiency of different methods of operating individual reservoirs.

R. H. O.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS.*

OPTIMUM TIME OF DELAY FOR PARACHUTE OPENING.

The opening of a parachute subjects the parachutist to a sudden and painful jerk, the severity of which depends approximately on the square of the speed at the time of opening. Openings at high speed may therefore result in immediate injury or tearing of the parachute.

As a result of calculations made by W. A. Wildhack of the Aeronautic Instruments Section, it has been shown theoretically that for horizontal launching, the velocity of a parachutist falling with parachute closed will pass through a minimum less than either the launching speed or the terminal speed. The terminal speed is that at which the air resistance equals the pull of gravity. It is reached after 10–12 seconds of fall, and is about 160–175 feet per second or 110–120 miles per hour at low altitudes. The best time to open the parachute with as little shock as possible is clearly the time of minimum speed.

In a report on this work, which will be published in the June number of the *Journal of Aeronautical Sciences*, formulas are derived relating the minimum velocity to the launching velocity and to the angle of the trajectory at which the minimum occurs. The time interval between the launching and the occurrence of minimum speed is determined from step-by-step computations for various launching velocities.

Charts are given for graphical determinations of the optimum time of delay, the velocity, and the distance fallen at any time before opening, for various launching velocities.

The formulas and charts are derived in general form, making them equally applicable to problems of landing material as well as personnel by parachute, given only the terminal velocity (or the mass and the drag at a given airspeed). As

*Communicated by the Director.

one example, values obtained for an average parachutist (terminal velocity 160 feet per second) launched with a horizontal velocity of 160 feet per second, indicate that a minimum speed of 125 feet per second will be reached after 2.8 seconds, when the direction of falling makes an angle of 35° with the horizontal, and the vertical distance fallen is about 120 feet. The opening shock for the optimum delay would be approximately the square of the quotient of 125 divided by 160, or 61 per cent. of what it would be for immediate opening, or for opening delayed more than 12 seconds. Even for a launching velocity of 500 feet per second, the opening shock may be reduced to 80 per cent. of the value for terminal velocity by opening the parachute at the proper time—after 7 seconds for the average parachutist.

It is shown that the effect of altitude is to lengthen the optimum time of delay for a given indicated air speed at launching, the sea-level value having to be multiplied by the square root of the density ratio.

THE BETA-ANOMALY IN RUBBER.

When Ruhemann and Simon measured the specific heat of rubber in 1928, they found three anomalies which they called the α, β, and γ. The existence of the α-anomaly, near $-70°$ C., has been confirmed by the later investigators. The γ-anomaly, found at temperatures between 15° and 30° C., was present only in rubber which could be shown by other means to be at least partially crystallized, and was said to be due to the melting of the crystals. There has been no question as to the correctness of this explanation.

The β-anomaly, however, which Ruhemann and Simon found near 0° C., has never been satisfactorily explained. Other observers have not been able to find irregularities in the specific heat or other properties of rubber in this region. Consequently, the existence and nature of the β-anomaly have remained a matter of doubt.

In a letter to the editor of the *Journal of Chemical Physics*, Lawrence A. Wood of the Bureau's Rubber Section points out that the results of Ruhemann and Simon can now be explained

as a phenomenon of crystallization without the assumption of an anomaly characteristic of rubber in general.

Recent work at the Bureau has shown that the melting range of crystalline rubber is dependent on the temperature at which crystallization has taken place. Crystallization is most rapid near −20° C., and it is probable that the experimental conditions of Ruhemann and Simon were such that the rubber was at a temperature near −20 C., for a sufficiently long period for crystallization to occur during their experiments. Rubber crystallized at such a temperature would be expected to melt near 0° C., and to give rise to a peak in the specific heat curve like the one which Ruhemann and Simon named the β-anomaly.

The conclusion may then be drawn that the anomaly found near 0° C. by Ruhemann and Simon represented merely the melting of crystals formed under the particular conditions of their experiments, and that this anomaly is not characteristic of rubber in general.

EFFECT OF ALTITUDE ON KNOCK RATING.

Since the knock rating of motor gasolines was placed on a uniform basis by the adoption of the ASTM Motor Method in 1933, results obtained by laboratories located at different altitudes have shown systematic differences. Attempts were made to eliminate the variation by increasing the volumetric efficiency of engines operating at higher altitudes so as to maintain constant absolute compression pressure. Some improvement followed this measure, but deviations were still beyond experimental error.

Adoption of the 1939 CFR Research Method of knock rating raised the problem again. Field tests in which altered operating conditions were tried showed no great improvement. The Bureau was asked to carry out coöperative tests in its altitude chamber to determine appropriate conditions for rating by each test method at altitudes up to 7,000 feet. In the *Journal of Research* for June (RP1475), Donald B. Brooks describes and gives the results of these tests. The first series, made on one engine, indicated that a linear change in cylinder clearance volume with air pressure gave the desired results,

the necessary rate of change being the same for all levels of octane number, but differing for the two test methods. Field tests verified the results. A second series using three additional cylinder assemblies, substantiated the findings and developed the proper relations between cylinder clearance volume and octane number for both test methods. This information was verified and extended by field tests.

Equations were then derived to relate cylinder clearance volume to octane number and to altitude. From these relations, equations were deduced to relate octane number requirement to altitude. By comparison with field tests, these were found to be of general applicability.

WEATHER RESISTANCE OF PORCELAIN ENAMEL ON IRON STRUCTURAL UNITS.

The recent increase in the use of porcelain enamel on architectural units, and the diminished availability of zinc and tin for galvanized roofing and siding have accentuated the need of data on the weathering resistance of this type of protective coating.

In 1939, W. N. Harrison and D. G. Moore began a study on the weathering properties of 14 types and colors of enamel, including glossy, semi-mat, and mat finishes in both acid-resisting and non-acid-resisting compositions. This study, which is still in progress, involves a total of 864 one-foot-square panels and an equal number of 4 by 6 inch laboratory specimens. Installation of specimens in racks for exposure at four selected locations (Washington, D. C., St. Louis, Mo., Lakeland, Fla., and Atlantic City, N. J.) was completed in 1940.

At the end of the first year of weathering, all panels were inspected. As reported in RP1476 in the *Journal of Research* for June, the changes in the enamel surface were, for the most part, slight. In most cases, comparison with a storage panel of like composition was necessary to render any changes which had occurred visible to the naked eye. Even where the effects of weathering were readily visible, changes were confined to the surface of the enamel only; no failure of the

enamel coating to protect the underlying metal from rusting was discovered.

The first year inspection resulted in the following observations: (1) The glossy, acid-resistant compositions were in excellent condition at all locations. (2) The non-acid-resisting enamels, as a whole, showed evidence of weathering. (3) The mat enamels appeared to be unsuited for architectural use where appearance is important, because of fading and the difficulty of cleaning them. (4) Some of the colored non-acid-resistant enamels were slightly faded after one year because of tiny pits on the surface, caused probably by acid-forming gases such as carbon dioxide and sulfur dioxide in the atmosphere.

The pitting of the non-acid-resistant enamels was produced in the laboratory by subjecting them, while immersed in water, to carbon dioxide under slight pressure. Subsequent thorough drying was necessary to bring out the pits.

FADING RATE OF PAINTS.

Ability to resist fading is a valued property of most paints and textiles. Two important evidences of fading—change of color and change of gloss—can now be measured by photoelectric methods developed within the past few years. In the June *Journal of Research* (RP1478), A. J. Eickhoff and R. S. Hunter describe a study in which photoelectric tristimulus measurements of color change and photoelectric measurements of specular-gloss change were used to follow the fading of several paint samples. These were exposed both outdoors and to two machine treatments (*A* and *B*) designed to weather the samples at an accelerated rate. Thus, it was possible to compare numerically the rates of artificial and natural fading of the paints. The data show: (1) The treatment used with apparatus *A* caused fading which averaged 20 times as fast as fading outdoors, but the speed-up factor varied from roughly 5 times for one paint to roughly 40 times for another; (2) the treatment used in conjunction with apparatus *B* caused fading which averaged 5 times as fast as fading outdoors, but the speed-up factor varied from roughly 3 times for one paint to roughly 20 times for another; and (3)

for almost every paint tested, the factor relating the speed of fading from treatment in apparatus *A* to the speed outdoors was more nearly constant through the whole fading process than the corresponding factor for treatment in apparatus *B*.

Thus, treatment *A* not only faded paints faster, but it provided a preview of the course of fading which was usually a better representation of outdoor fading than that provided by treatment *B*. The data collected during the study are noteworthy chiefly for the methods they demonstrate. These should be valuable for future studies of the fading of materials and for the examination of methods for accelerated fading.

DETERMINATION OF CARBON AND HYDROGEN IN BONE BLACK AND OTHER CHARS.

The ability of bone black and a variety of other materials of varying carbon content to adsorb large quantities of gases and to remove colored substances from solution indicates that carbon itself is the principal seat of the adsorptive activity. Among the carbon-containing substances available to industry, the content of carbon varies from approximately 5 per cent. in some bone blacks to 90 per cent. in some vegetable and other activated carbons. It is believed that two general factors determine the extent of adsorption or "activity," namely (1) the quantity of carbon present, and (2) the specific surface and physical state of the carbon surface. It is, therefore, important to have accurate data on the percentage of carbon in such materials as a necessary part in a fundamental study of the chemical and physical properties of bone black and analogous materials. Such data are presented in the June *Journal of Research* (RP1479) by Victor R. Deitz and Leland F. Gleysteen, Research Associates, working on a joint research project undertaken by the United States Cane Sugar Refiners and Bone Char Manufacturers and the Bureau.

The procedure for the special handling of such highly adsorptive substances is set forth with a description of the necessary apparatus. It is not possible to dry these materials to a constant weight in the conventional laboratory oven, nor is the use of a vacuum oven recommended because of dusting of the material. Instead, a simple procedure has been adopted

to bring each sample for analysis to a constant weight by exposing it to water vapor in an exsiccator for 18 hours, and subsequent heating in a helium atmosphere at 105° C. for 18 hours. Reproducible results are obtained by the combustion in oxygen of samples treated in this manner.

The samples are analyzed for carbon and hydrogen by complete combustion in oxygen with the weighing of the resultant carbon dioxide and water. Auxiliary determinations are made of the carbonate in the ash from the combustion and of the carbonate and adsorbed carbon dioxide contained in the original material. The results are tabulated to give these separate contributions to the percentage of carbon.

A comparison was made with results for the same materials obtained by determining the loss upon ignition of the carbon residue from acid washing. The results indicate considerable deviation, which is attributed to volatile hydrogen, nitrogen, sulfur, and oxygen compounds which are not removed by acid washing. Although the data obtained by the combustion-in-oxygen method are to be preferred for use in research work and in other cases where accuracy is required, it should be pointed out that the data obtained by the loss-upon-ignition method are undoubtedly adequate for control purposes in the general routine of a sugar refinery laboratory where such data are of an empirical nature.

CORROSION IN SOILS.

Letter Circular LC689, "Corrosion in Soils," recently released, is a summary of data on a considerable number of phases of underground corrosion. The paper which was prepared by Kirk H. Logan, chief of the Bureau's Underground Corrosion Section, is intended to save letter writing in answering oft-repeated questions regarding corrosion, many of which cannot be answered briefly because of the number of factors involved.

The field of underground corrosion and its mitigation is covered briefly and nontechnically in most of its aspects, and numerous references are given in support of statements, or as supplementary reading. The paper is so arranged that any one with a general knowledge of corrosion phenomena need

read only that section in which he is particularly interested. The paper discusses briefly the theory of corrosion, soil characteristics, causes of corrosion, soil tests, and methods of mitigating corrosion, including the selection of pipe materials, protective coatings, and cathodic protection. Underground corrosion is frequently the resultant of factors, some of which are characteristics of soils, while others arise from methods of installing pipe lines.

THE FRANKLIN INSTITUTE.

LIBRARY NOTES.

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. Orders received in the morning are filled the same day. The average cost for a print 9 × 14 inches is thirty-five cents.

The summer war schedule of the library, now in effect, is as follows: Mondays, Tuesdays, Wednesdays and Fridays, from 9 o'clock A.M. to 5 o'clock P.M.; Thursdays, from 9 o'clock A.M. to 10 o'clock P.M.; and Saturdays until 12 o'clock noon.

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NOTES FROM THE BARTOL RESEARCH FOUNDATION

CLOUD-CHAMBER TRACK OF A MESOTRON STOPPED BY GAS*

BY

T. H. JOHNSON AND R. P. SHUTT

In the cloud chamber arrangement previously described [1, 2] the picture (Fig. 1) has been obtained which shows a mesotron track ending in argon of 1.3 atmospheres. The track, curved by a magnetic field of 1,150 oersteds, shows the particle entering a lead plate of 1-cm. thickness from which it emerges heavily ionizing. It underoges 2 single scatterings of about 3° each at A and at B, and it ceases to ionize at C. The approximate mass of the particle has been determined in two ways. First, the radius of curvature of the track above the lead plate is 150 ± 50 cm. Combining this with the fact that its range is approximately 1 cm of lead we find a value of $m = 100 \pm 50$ electron masses. Below the lead plate the radius of curvature between A and B is $\rho_1 = 23$ cm., and between B and C it is $\rho_2 = 17$ cm. The total ranges at the corresponding positions are, respectively, $R_1 = 12.0$ cm. and $R_2 = 6.4$ cm., from which we find $m_1 = 80$, and $m_2 = 70$. For the computations of m the energy-momentum curves published recently by Rossi and Greisen [3] have been used.

The rather large curvatures in the second compartment can be measured with considerable accuracy, but errors are introduced by the multiple scattering which produces additional curvatures superimposed upon those caused by the magnetic field. Williams [4] has deduced the *average* radius of curvature ρ_s owing to the multiple scattering near the end of a track and finds

$$\rho_s \approx R \times 1.3(m/Z)^{\frac{1}{2}},$$

* Reprinted from The Physical Review 61, 380 (1942).

[1] T. H. Johnson, J. G. Barry, and R. P. Shutt, *Phys. Rev.*, **59**, 470 (1941).

[2] R. P. Shutt, *Phys. Rev.*, **61**, 6 (1942).

[3] B. Rossi and K. Greisen, *Rev. Mod. Phys.*, **13**, 240 (1941).

[4] E. J. Williams, *Phys. Rev.*, **58**, 292 (1940).

where R is range of the particle, m is its mass, and Z is the atomic number of the scattering gas. As ρ_s may be either positive or negative one has to write

$$1/\rho - 1/\rho_s < 1/\rho_p < 1/\rho + 1/\rho_s,$$

where ρ_p stands for all probable values of the radius of curvature caused by the magnetic field alone. In our case one finds 11 cm $< \rho_{p1} <$ 42 cm. and 6 cm. $< \rho_{p2} <$ 39 cm. These give $m_{p1} = 80$ with an uncertainty of $(+ 105, - 55)$

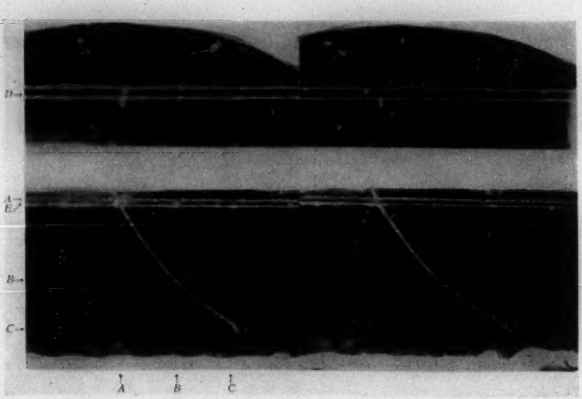

FIG. 1. Stereoscopic photographs of negatively charged mesotron stopping in the gas of the cloud chamber. Between the two compartments there is a lead plate 1 cm. thick. D and E indicate scales aiding in the exact determination of geometrical positions.

and $m_{p2} = 70(+ 150, - 55)$. Averaging over m_1 and m_2, we have $m = 75(+ 90, - 40)$. This value is lower than that usually found, but this may arise from the inaccuracy introduced by the multiple scattering.

The direction of curvature is that of a negatively charged particle. It is also noted that no disintegration electron appears near C. The track ends at a distance of 1.4 cm. in front of the background velvet. Near B it goes out of the directly illuminated part of the chamber and receives its light

only from numerous mirrors placed around and inside the chamber, but by comparison with other pictures taken in the same arrangement a track of low density such as that of a fast electron would still be visible as close as 0.5 cm. to the velvet. A mesotron at the point where it stops ionizing still has an energy of between 10^3 and 10^4 ev. or a velocity of about 5×10^8 cm. per sec. Since the particle appears not to disintegrate within a sphere of 0.9-cm. radius around the end of the track, its lifetime after it had ceased to ionize must have

TABLE I.

Data on Mesotrons Stopped in Gas of Cloud Chambers.

Reference	Number of electron masses	Charge	Electron track appearing near end of mesotron track	Remarks
a	220±35	+	slight indication	good, stereoscopic pictures
b	170±9	−	no	doubtful, non-ster. pict.
c	20?	+		very doubtful,
	100±30	+		stereosc. pictures
	120±30	−	no	
	55±35	−		
	170±100	+		
d	250±70	+	yes	good, stereosc. pict.
e	not determined		yes(?)	somewhat doubtful,
	not determined		yes(??)	stereosc. pict.
This article	75(+90, −40)	−	no	stereoscopic pictures

[a] S. H. Neddermeyer and C. D. Anderson, *Phys. Rev.*, **54**, 88 (1938); Rev. Mod. Phys. **11**, 191 (1939).

[b] Y. Nishina, M. Takeuchi, and T. Ichimiya, *Phys. Rev.*, **55**, 585 (1939).

[c] H. Maier-Leibnitz, *Zeits. f. Physik*, **112**, 569 (1939).

[d] E. J. Williams and G. E. Roberts, *Nature*, **145**, 102 (1940).

[e] E. J. Williams and G. R. Evans, *Nature*, **145**, 818 (1940).

been greater than 10^{-8} sec., with allowance made for a considerable increase of its path by scattering. Because of the high residual velocity of a mesotron after it stops losing energy by ionization, one cannot expect to find a disintegration electron near the end of every mesotron track. Table I shows a record of all published photographs of mesotrons stopping in the gas of a cloud chamber. Reviewing the records it appears that disintegration electrons have only been found from positive mesotrons.

Concerning the frequency of ends of mesotron tracks occurring in the gas of a cloud chamber we should expect according to Williams [5] to find 8×10^{-7} particles per cm. With one such track found among 42,000 pictures in a cloud chamber containing 50 cm. of gaseous path we actually find 5×10^{-7} track ends per cm.

The authors are indebted to the Carnegie Institution of Washington for extensive support given to these researches. They also wish to thank Mr. M. H. Hornstine and Mr. W. C. Sheppard for their assistance in operating the cloud chamber.

[5] E. J. Williams, *Proc. Roy. Soc.*, **A172**, 194 (1939).

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION.

The Use of Radioactive Substances in Medicine.—Dr. V. W. Murray Wright, Director of Clinical Research of the Biochemical Research Foundation, on May 15, 1942, gave a seminar before the Medical Staff of the Naval Hospital at the League Island Navy Yard, Philadelphia, Pennsylvania. He outlined his experiences during the month of August (1941) while studying at the Radiation Laboratory in Berkeley, California, and his treatment of some 15 patients in Philadelphia with radioactive phosphorus produced by the cyclotron of the Biochemical Research Foundation at Newark, Delaware.

Radioactive chemicals are increasing the medical knowledge of normal physiological processes in the animal body and also are aiding in the fight against pathological conditions produced by disease. Perhaps the two most striking examples of the latter are the control of the chronic leukemias (so-called cancer of the blood or too many white blood cells) and polycythemia vera (too many red blood cells). Excellent results have been obtained in treating these two conditions by occasional injections into a vein in the arm of small amounts of a solution of radioactive Na_2HPO_4.

This method of treatment actually supplies *internal* radiation to the diseased tissues instead of *external* radiation as formerly achieved with powerful X-ray machines. The latter treatment often causes "X-ray sickness" and a temporary anemia whereas radioactive phosphorus causes no reactions and improves any associated anemia. Treatment with radioactive phosphorus is of no value in cases of acute leukemia which live but a short time.

Many chronic cases of leukemia, treated at Berkeley, California, are still well and happy three years after the initiation of this type of treatment, requiring only an occasional injection.

Formerly individuals suffering with polycythemia vera

were treated by periodic venisection (so-called "blood-letting" or "bleeding") or by the use of phenylhydrazine. The use of radioactive phosphorus in this condition has proven to be superior to either of the former methods. In the opinion of Dr. Wright, radioactive phosphorus does not cure either chronic leukemia or polycythemia vera but controls either process much better than previous forms of treatment. He likens its use to that of insulin which though it does not cure diabetes, yet is the most potent weapon thus far found for combating this disease.

Radioactive phosphorus has likewise been found very beneficial in early case of Hodgkin's disease, reticulo-cell sarcoma and in lympho-sarcoma.

Radioactive iodine is now being given (at Berkeley, California) as a "cocktail" or intravenously to patients suffering from diseases of the thyroid gland (goiter). In the past, Lugol's solution containing potassium iodide and iondine, has been given orally for such conditions and occasionally external radiation also by X-ray machines. As radioactive iodine when taken into the body is quickly and selectively absorbed by the thyroid gland, this newer form of treatment supplies both iodine to the deficient gland and *internal* radiation.

As radioactive strontium is largely absorbed by or deposited in the skeletal system it is being used in the treatment of various cancers of bone such as osteogenic sarcoma, periosteal sarcoma, multiple myeloma and metastatic carcinoma. The final results in treating these difficult conditions remain to be determined.

A serendipitous result of the use of radioactive strontium in several California patients (who had various forms of cancer) was that skin diseases from which they also suffered (such as psoriasis and Kaposi's disease) were incidentally cured. Though the cause of psoriasis and Kaposi's disease has never been found, the cure or control fortunately has.

The war, unfortunately, has interrupted and will continue to do so, much scientific research while accelerating other research to better prosecute defense. There seems to be little doubt of the possibilities of new and greater medical

discoveries when, with peace, there will be increased opportunities to obtain and use radiactive substances. The future along such lines is bright with promise.

Invert Sugar as a Substitute for Glucose in Intravenous Therapy.—S. E. KERR AND R. J. PAULY. This work was done in the Division of Pharmacy, American University of Beirut, Beirut, Republic of Lebanon, where the preparation of invert sugar was elaborated for use in the Hospitals of the American University. (*Surgery, Gynecology and Obstetrics*, **74**: 925, 1942.) Glucose being unavailable in the Lebanon because of blockade, invert sugar was prepared by a simple procedure and used in over two thousand cases with no unfavorable reaction. Commercial cane sugar in hot concentrated solution is split to its constituent molecules of glucose and levulose by addition of hydrochloric acid, followed by exact neutralization with sodium hydroxide within one minute. After filtration and adjustment of the acidity to a pH within the range 4.5 to 6, the stock solution is diluted and sterilized for use. The resulting solution diluted to a concentration of five per cent. hexose is a clear, colorless liquid containing not more than 0.01 per cent. of sodium chloride and 0.05 per cent. of unhydrolyzed sucrose. Studies on experimental subjects revealed that the levulose fraction disappears rapidly from the blood stream, the maximum value observed being 18 mg. of levulose per 100 cc. of blood at the termination of an infusion of 500 cc. of 10 per cent. invert sugar.

BOOK REVIEWS.

THE PHYSICAL EXAMINATION OF METALS, by Bruce Chalmers and A. G. Quarrell. Volume II, electrical methods, 280 pages, illustrations, 15 × 22 cms. New York, Longmans, Green & Company, 1942. Price $6.00.

No one needs to be reminded of the importance of this subject to industry, and the progress made in recent years. Present knowledge of metals has come, very largely, from the application of scientific methods to their investigation. The many different methods in use have resulted in two conditions, (1) that there is a lack of familiarity with the limitations, purposes, and accuracy of some of the methods by those who may profit thereby, and (2) that there exists a feeling among many, who apply the methods, that certain of them are better than others, which condition is due to greater knowledge of some than others. Undoubtedly, a clarification of this matter will further metallurgical technique.

The two volumes, at hand, comprise a set. Volume one treats on optical methods of examination of metals. It gives a brief introduction on the main properties of light, and proceeds through geometrical optics, wave optics (which is divided into parts on interference and on diffraction), polarised light, and a final chapter on sources of light.

Volume two is devoted to electrical methods. Under the heading of magnetism, methods are described for testing irregularities, detecting flaws, and other purposes. Electrical measurements are treated, on including the use of thermoelectric effects to study metals, and electrical methods of measurement of thickness from one side. A very interesting and well-arranged chapter is that devoted to x-ray diffraction. There is described the Finch electron diffraction camera, and considerable attention is given to the electron microscope and the subject of radiography. Details of the process of electrolytic polishing, as applied to a number of metals, are given in Appendix I, while Appendix II consists of various x-ray data that are useful for reference.

The books give adequate explanation of the theory underlying the physical examination of metals together with a description of the more important applications that have been made, and to a limited extent, a description of the technique. They are valuable to all those having to do with the examination of metals, and their use should encourage the employment of suitable and best methods.

R. H. OPPERMANN.

A MANUAL OF AIRBRUSH TECHNIQUE, by J. Carroll Tobias. 164 pages, illustrations, 18 × 26 cms. Boston, American Photographic Publishing Company, 1941. Price $3.00.

The airbrush is an instrument designed to convey paint or color from a supply reservoir to the work, in a very finely divided spray, by means of air pressure. It is a tool which takes its place alongside of the brush, the pen, and the pencil, as a tool for art work. Effects, impossible to achieve by any other means, are possible with the airbrush; but, to secure those effects, to take full

advantage of the power of the airbrush, the student must school himself by study and practice in airbrush technique. This book aims to give such instruction, and in addition, to show some of the many diverse applications and their results.

A rather thorough description is given of the instrument before actual handling, and lessons in its use appear. Then, specific uses are covered; such as, retouching negatives, coloring large prints, making and eliminating backgrounds, wash drawings, free-hand drawing, illustrations, monumental designing, portraits, and fashion illustrations. The uses are many and the results surprising, as shown by photos in the book.

Artists and photographers, as well as anyone who has a desire to produce artistic work, will find here an interesting, instructive book on a fascinating instrument.

R. H. OPPERMANN.

THE HIGH-SPEED COMPRESSION-IGNITION ENGINE, by C. B. Dicksee. 331 pages, illustrations, 16 × 22 cms. New York, Interscience Publishers, Inc., 1940. Price $4.50.

This book is written from the British standpoint with regard to conditions of economics, as affecting the development of this type of engine as a means for transport. The imposition of a tax on petrol in Great Britain encouraged the development of the high speed compression-ignition engine, as this offered considerable saving over the heavy, slow speed engine. Today, the former has almost completely ousted the latter from the heavier forms of road transport. The book at hand deals with the principles governing the operation of these engines, and discusses the problems associated therewith.

It opens with a brief discussion on the laws of gases to the extent necessary for an understanding of the internal combustion engine. Then, what is called "idealized cycles of operation" are discussed. These are three in number—(1) the constant volume cycle, (2) the constant pressure cycle, and (3) the mixed cycle which is a combination of the other two. The various conditions affecting the operation of the engine are next taken up, including the losses and limitations of the practical engine, the air charge before the admission of fuel, the process of combustion, air movement, types of combustion chamber, and fuel injection. And finally, reference is made to some practical results, such as the variation of efficiency with load, heat balance, and a forecast of future developments.

Although written expressly for British consumption, the treatment here given will be of use to all those interested in the subject. It is clear and conclusive.

R. H. OPPERMANN.

ELECTRIC MOTORS IN INDUSTRY, by D. R. Shoults and C. J. Rife. Edited by T. C. Johnson, 389 pages, illustrations, 16 × 24 cms. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1942. Price $4.00.

The proper motoring of industrial plants has always been an important matter, even to the point of being a specialty. Yet, there never has been a time when this has been observed, universally. Always, there has been someone who has disregarded the characteristics of load and of motors, so that today it is not at all difficult to find overmotoring and similar evils. Probably one reason for this

is that experience plays so large a part in proper application, and texts on the subject only included obvious principles of electrical engineering. This is still true. The book at hand, however, directs those with engineering knowledge into specially selected studies on the subject, eliminating all that is unnecessary, and going as far as possible in presenting the experience of others.

The opening chapters deal with industrial power supply, and in the interest of completeness, make brief and direct reference to comparisons of purchased and locally generated power. The important subject of power factor is here discussed as well as some of the problems of bringing electrical power to the point of use. After this, the fundamental principles and limitations in motor design are reviewed together with the elements of mechanics necessary for an understanding of the loads on which the motor torques are exerted. The induction motor is given much attention, its characteristics and its applications. Similarly synchronous apparatus is discussed, then direct current motors. The study of motor control furnishes the link between the power supply and the motor itself, and the last topic necessary to the determination of the type of motor to use in certain cases. There remains the determination of the proper size motor for a given application, and this is taken up at this point. Examples are then given of the application of various drive systems to special purposes.

The last two chapters are devoted to description and use of electronic devices in industry. While considerable use has been made of such devices, and many engineers are somewhat familiar with them, the field of application is new, and the rather general treatment here given is not a waste of time.

A feature of the presentation is the so-called "physical picture" which gives a sense of proportion and fitness. The induction motor is thought of in terms of a slipping clutch, rather than in terms of an equivalent circuit, or complicated equation. This makes for deeper impressions. At the end of each chapter, there is a list of problems, some of which require calculations for their solutions. The answers for these are not given. Some criticism may be directed here from the standpoint of the use of the book for home study, particularly by those who may not be familiar with the theory or formulae.

The book contains a great amount of information, arranged and presented in a good style. It is a combination of theory and the results of practice that is valuable to industrial engineers whose interests be in motor application, to equipment sales engineers, and to power salesmen of utility companies.

R. H. OPPERMANN.

A SHORT COURSE IN TENSOR ANALYSIS, by Gabriel Kron. For Electrical Engineers, 250 pages, 16 × 24 cms. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd., 1942. Price $4.50.

Gabriel Kron needs no introduction to those interested in the solution of equations of performance, of complex engineering problems, in an organized manner. His work has appeared previously in book form and periodical articles. The present volume, containing a series of lectures delivered to students in the Advanced Course in Engineering of the General Electric Company, represents a short outline of the tensorial method of reasoning that may be used to advantage in the analysis of engineering problems. The subject matter has been selected from the point of view of the power engineer and is divided into two parts.

The introductory concepts, of matrices and matrix manipulations, are first presented, then the transformation theory of tensors is developed with the concept of the "primitive network." Interconnection of coils and networks is illustrated by multiwinding transformers, armature windings, and mercury-arc rectifier circuits. The study of phase-shift transformers brings out the hypothetical reference frame of Fortescue. Differentiation of tensors are shown by the field equations of Maxwell.

The second part opens with a treatment on generalization postulates, which assists in the development of the transient and steady state equations of rotating electrical machinery. In this, a direct current machine with two sets of brushes is used as a primitive of all synchronous and induction machines. Stability of regulating devices, the revolving field theory, speed control systems, the hunting of machines, and the equation of motion are additional topics.

The treatment is a rigorous, well presented exposition of tensor analysis of electrical engineering problems. Great care is taken in the selection and development of concepts to include only those that form the foundation of theoretical physics.

R. H. OPPERMANN.

PUBLICATIONS RECEIVED.

The Recovery of Vapors, with special reference to volatile solvents, by Clark Shove Robinson. 273 pages, illustrations, 16 × 24 cms. New York, Reinhold Publishing Corporation, 1942. Price $4.75.

The "Particles" of Modern Physics, by J. D. Stranathan, Ph.D. 571 pages, illustrations, 16 × 24 cms. Philadelphia, The Blakiston Company, 1942. Price $4.00.

Chemistry and Physiology of the Vitamins, by H. R. Rosenberg, Sc.D. 674 pages, illustrations, 16 × 24 cms. New York, Interscience Publishers, Inc., 1942. Price $12.00.

Foundations of Astronomy, by W. M. Smart. 268 pages, illustrations, 15 × 23 cms. London, New York, Toronto; Longmans, Green and Co., 1942. Price $4.20.

Essentials of Astronomy, by John Charles Duncan. 181 pages, illustrations, 15 × 23 cms. New York and London, Harper & Brothers, 1942. Price $1.85.

Introduction to Physics, by Harley Howe. 559 pages, illustrations, 16 × 24 cms. New York and London, McGraw-Hill Book Company, Inc., 1942. Price $3.75.

CURRENT TOPICS.

Texans Cited for Speed in Building Bomber Plant.—(*Refrigerating Engineering*, Vol. 43, No. 4.) More than 8000 Texans received merit award citations from The Austin Company recently in recognition of their part in speeding completion of a large windowless, air-conditioned bomber plant. Built of fiberglas and steel to provide maximum safety, automatic blackout protection, economical air conditioning, this plant and an identical structure several hundred miles away are the two largest air-conditioned buildings in the world. Faced with the need for uninterrupted production of heavy bombers at the fastest possible rate, the Army Corps of Engineers and Air Corps set standards which would be sure to maintain working comfort and high production efficiency on their assembly lines right through the sultriest summer heat. The "ferroglas" construction introduced by Austin Engineers to meet these requirements combines glass fiber, sheet steel products, special vapor proof paper and asphalt coatings in what are probably the most heavily insulated side walls and roofs ever designed for a factory. It also gives the plants exceptional sound control, since at least 75 per cent. of all factory noises will be absorbed in the walls and ceilings. The plant is lighted entirely by two types of fluorescent lamps, and has white cement floors which reflect light upon the underside of the wings and fuselages to aid men working underneath the bombers which move them down a long, unobstructed assembly line extending the entire length of the structure. A special wiring system worked out for these plants reduced by one third the amount of copper required for the lighting system.

R. H. O.

Floor Coils Heat Auditorium of New Community Church.—(*Heating, Piping, and Air Conditioning*, Vol. 14, No. 4.) FRANK LLOYD WRIGHT prepared the plans for the new thin-walled steel and gunite building of the Community Church in Kansas City, which is a structure apart from its fellows. From its heating and air conditioning system to its flat roof and shallow crown from which searchlight beams eventually will slice the heavens with a spire of light, it is different. Seating arrangements are similar to those of an amphitheater. Rows of theater seats are on steps that rise from the pulpit platform to the rear of the auditorium. The aisles

are gently sloping ramps. In this church, gunite comes into its own as a primary building material. Shot under pressure, it has formed the walls, ceilings and floors. The walls are about 3 inches thick. They were built about a "bird cage" of 2 inch square steel tubing. Onto that went paperbacked mesh and then the layers of gunite leaving about a 2 inch insulating air space between the inner and outer walls. The church has no windows, except those in the offices. At the present time there still are two units to be built, a chapel and auditorium to seat about 250 persons, a kitchen, dining rooms and additional classrooms; and extensive parking levels and ramps. Because of the war, those units will remain in the plan stage for the present. Beneath the floor of the 1200 seat auditorium, with its upholstered individual theater chairs, run coils of welded wrought iron pipe, through which hot water supplied by an oil burner is circulated. The piping is spaced on the unit lines of the building. The supply pipes pass beneath the center of the seats, the cooler return pipes beneath the feet of the worshippers. The auditorium of the church is heated by this floor panel which consists of 3600 ft of 2 in. welded wrought iron pipe coils on 16½ in. centers connected to 2½ in. supply and return headers in the aisles. In summer, chilled water is to be circulated through these floor coils for precooling. For other rooms in the structure there are factory built air conditioning units served with hot water in winter and chilled water in summer from a 30 hp. condensing unit.

R. H. O.

Domed Building Fitted to Research Needs.—(*Engineering News-Record*, Vol. 128, No. 15.) The design of a suitable structure for housing the hundred million volt cyclotron now under construction at the University of California, involved considerable pioneer work, not only because of the unusual shape and size of the structure but because there was no advance knowledge of just how requirements may change as research progresses with this unusual piece of apparatus. Obsolescence might even affect the usefulness of the structure for this particular purpose in a relatively short time. It was therefore desirable to avoid any unnecessary expense in the structure itself and to use materials with a maximum salvage value. Beginning with the idea of a building octagonal in plan, numerous variations were studied until a satisfactory solution was found in the interesting dome type structure. The decision was reached to locate the building on the shoulder of a remote high hill where the desired encircling clearance was available. Control rooms for the operators are to be located beyond the brow of the

hill, thus interposing an earth shield or screen between them and the cyclotron. Clear floor space is needed for 80 feet radially from the center of the cyclotron and all of this space is required to be under a crane of 30 ton capacity. These requirements are most satisfactorily met by a 24-sided building. As all of the sides of such a building are equidistant from the cyclotron, it will be easy to add the wings that may later have to be located along radial lines extending from the center of the atom smashing instrument. The roof framing chosen consists of 24 ribs or trusses all connected to a drum at the center or crown of the dome and tied together at the spring line by a polygonal steel tie. The lower end of each truss is carried by a wall column. The dome is therefore virtually an aggregation of crescent trusses with the drum at the crown forming their common connection. Horizontal movement at the bottom is restrained by a steel tie, a 16WF40 beam placed with the web horizontal. Elongation of this beam under load is taken into account in the design.

R. H. O.

Harvard Air Conditions Rare Book Library.—(*Heating and Ventilating*, Vol. 39, No. 4.) Some of the rarest and most valuable books and manuscripts in the world, a group conservatively valued at a million dollars, are now safely housed in an air-conditioned library at Harvard University. These are part of a collection of some 90,000 pieces, previously stored in the famous Treasure Room at the Widener Library, alongside which the new building has been erected. The structure, three stories above ground, and three below ground level, is the gift of Arthur Amory Houghton, Jr. of Corning, N. Y., rare book curator of the Library of Congress. The building is of Georgian style architecture, brick and limestone, matching other important buildings in Harvard Yard. The thick walls are waterproofed and insulated with 1 in. of cork. Double-glazed windows and glass wool in the roof structure give additional insulation. Included in the collection is the only book from the 5,000 volume library of John Harvard that was saved from the fire of 1765, which destroyed most of his collection and badly damaged Harvard Hall. About 400 volumes of the John Harvard library had been out on loan and so were not in the fire. The air conditioning equipment includes an electrostatic air cleaner, spray chamber with misting nozzles, a preheating coil, direct expansion cooling coils, heating coils, odor absorbers and a 30-ton Freon-12 compressor. Automatic controls are of the pneumatic type. Well water at 55° F. supplies the pre-cooling coil. Design calls for summer temperature

of 78° to 80° F. maximum dry bulb, with 45 per cent. relative humidity, and an allowed variation of 5 per cent.

R. H. O.

New Type Plasma Filter Will Aid in Prevention of Fatal Wound Shock.—Recourse to one of the new products of American Industry—glass in soft flexible fiber form—has effected an important improvement in the performance of the apparatus employed to give blood plasma transfusions to soldiers wounded in battle. A short strip of tape, woven of the new glass fibers, is inserted in the flexible tubing of the apparatus to filter out of the blood plasma any foreign particles which might get into it, and which, if they entered the blood stream, might cause serious complications. Military medicine is relying heavily upon blood plasma transfusions to save the lives of many soldiers that in past wars would inevitably have died from shock following upon their wounds. Shock occurs when damage to human tissue permits the fluid portion of the blood, with its proteins, to leak out from the fine blood vessels into the tissue. If the leakage continues until there is not enough fluid left to circulate the red blood cells that carry life-giving oxygen, the brain, kidneys, heart and other vital organs cease to function, and death ensues. Throughout the history of modern war shock has been responsible for the death of more soliders able to reach first-aid stations than any other cause, but this loss of fluid, or plasma, can now be corrected by the injection of human plasma which has previously been collected, prepared in proper form, and stored for use when required. The injected plasma replaces the lost fluid, enabling the oxygen-carrying red cells of the blood to resume transporting the oxygen without which life is impossible. Prior to the development of the new glass fiber filter, filters made of a number of different materials were tried out in the transfusion apparatus. A fine wire mesh of stainless steel proved unsuitable because it required bulky housing. The fibers of gauze filters swelled when wet, slowing down or stopping the passage of the fluid. The close weave of the Fiberglas filters effectively blocks even minute particles but permits free passage of the fluid since, being glass, the fibers do not swell under moisture. The Fiberglas filters are sterile, and can be resterilized for repeated use. The glass fiber filter represents, of course, only one of the many technical devices and methods which have made it possible to use blood plasma transfusions freely and effectively in military medicine. The sum total of these advances has given medicine a weapon that fights death at its source, and one that can be employed close up to the battle lines. Plasma,

unlike whole blood, does not have to be kept under special temperatures. After the red and white blood cells have been removed from human blood, the remaining fluid—the plasma—can be dried and preserved in that state. With ordinary care the plasma will retain its life-giving properties for several years. Just prior to the injection, the dried plasma is mixed with sterile water. Plasma, sterile water and the injection apparatus are packed together in easily handled kits, ready for almost instant use. Another advantage possessed by plasma over whole blood is that it does not have to be "typed" with the patient's blood.

R. H. O.

Magnetic Delay.—(*General Electric Review*, Vol. 45, No. 4.) A steel strip that, when placed under a powerful magnet, waits about a minute and a half before it jumps toward it, is used by Dr. Herbert H. Uhlig, of the G. E. Research Laboratory, to show a delayed change in the steel's atomic arrangement. Knowledge of the nature of such changes is important in making magnetic materials for transformers and other electric equipment. The strip is a common type of stainless steel containing 18 per cent chromium, 8 per cent nickel, and iron. This metal, known as 18-8, is not ordinarily attracted by a magnet, because it usually contains a little nitrogen. This stabilizes the nonmagnetic condition of the metal that occurs at temperatures above 147° F., and is reached as the steel is prepared. The nitrogen seems to cause friction, which prevents the atoms from rearranging themselves to the phase in which a magnet causes attraction. If a piece of the nitrogen-free metal is heated to 1100° F., then cooled, the magnetic phase does not return until it is down to the boiling point of water, 212° F. Dr. Uhlig found that if the heated strip is suddenly cooled by quenching in water, then placed under the magnet, it is not attracted, even though it is cooler than the temperature at which the change should occur. After about a minute and a half, however, it suddenly jumps to the magnet. Dr. Uhlig says that the atoms start rearranging themselves as soon as it is cooled, but it takes this time before a majority are shifted and at least a day before all reach a state of equilibrium. The change is one in the crystalline structure of the metal. At high temperatures, when nonmagnetic, it has the face centered arrangement, with the atoms forming cubes piled like bricks. The low temperature magnetic phase has them in the body centered arrangement, with cubes interlocking, so an atom in the center of one forms the corner of adjacent cubes. "From these experiments," said Dr. Uhlig, "we hope to learn more about

the heat treatment of steels and the preparation of alloys with better mechanical properties and corrosion resistance." He demonstrated the magnetic experiment recently before a meeting of the Schenectady Chapter of the American Chemical Society.

R. H. O.

Photography Utilized to Break an Aircraft Bottleneck.—The big bottleneck in airplane manufacture has been the loft, the department in which the draftsmen's renderings of construction details are enlarged into full scale drawings on huge sheets of plywood or metal. Layout men use the loft floor as their drawing board and crawl around on their drawings as they work. The practice, handed down from that far time when magnificient clipper ships were born in New England lofts, was carried over virtually intact into the aviation industry. It served its purpose well—as long as demand for airplanes was relatively small. Under leisurely schedules of peace time aviation manufacture, lofts could turn out enough full-scale drawings and templates made from such drawings to keep productive departments running. But increased war time demands turned lofts into bottlenecks. As demands accelerated, it became necessary to call upon the automobile industry to supply parts and sub-assemblies for fighters and bombers. These parts and sub-assemblies, made in scattered automobile plants, had to fit with absolute accuracy. To get such close fits the needs for the number of templates were multiplied beyond the combined capacities of all the lofts to supply them. At that crucial juncture the solution of the precision-loving pattern maker's problem was applied to the new problem. Now automotive and aviation engineers are lopping incalculable numbers of precious hours out of the once laborious job. Briefly the process works as follows: The original drawing is made on a sheet treated with phosphorescent chemicals which glow with blue light after bombardment by X-rays. A coating of cellulose nitrate lacquer now goes on the metal sheets which are to serve as working drawings or which are to be cut out and drilled as templates. Under red light the coated sheets are then faced with photo-sensitive film. This film having a nitrate skin, is locked to the surface of the metal by use of nitrate cement and pressure of rubber rollers. The phosphorescent drawing is then placed in contact with the photo-sensitive metal and the image is transferred. The exposed metal sheet goes successively through the developer, stop bath, hypo, wash tank and drying racks. With the immediate use of glass plates the resultant prints can be made positive and negative, as right and left templates. After close checking for fidelity,

the photographs on metal are easily cut into templates for specific production operations. With the use of photo-templates, accuracy is being held to tolerances of 1/1000th inch per foot, and as much as 250 hours of checking time has been eliminated in the production of a set of templates made from one drawing.

R. H. O.

Insect Extinguisher Patented.—By using a liquefied gas as a "carrier" of poison spray material, U. S. Department of Agriculture entomologists find that it is possible to divide the poison into so much smaller particles that it does not settle so quickly as it does when it goes into the air riding even the fine droplets of a liquid that forms a misty spray. Because some of the best insect poisons are deadly to insects even in inconceivably small doses, there is a gain in effectiveness against such flying pests as mosquitoes, flies, and gnats, from use of these gas-borne doses of poison. The poison particles stay longer in the air where the insects are flying and do not settle on the floor and walls as sprays of liquid do. This is the secret of the "aerosol" method of poisoning insect pests on which the Department's entomologists have just devised (and patented for public use) a new improvement. The first aerosol experiments involved spraying a volatile liquid (alcohol, for example) on a hot plate. The poison was dissolved in the liquid and was carried into the air on the gas that formed when the liquid was heated. The improvement comes from making use of one of several chemicals that have such a low "boiling point" that they are gases at ordinary temperatures and are liquids only when under pressure. Several of these are in common use as refrigerants. By first loading a cylinder with a minute dose of such a poison as nicotine, pyrethrum extract, or rotenone, and then filling the cylinder with the liquefied gas under pressure, the scientists have devised equipment resembling a common type of fire extinguisher that will spray out the liquefied gas under its own pressure. The gas will carry the poison with it in a form highly effective against insects. A suitable nozzle and a gas cut-off device make it possible to use the filled cylinder repeatedly until the liquid is exhausted. By using a gas that is not poisonous to humans there is the prospect that it will be practical to fumigate while a room is occupied. All the gas carriers have the advantage over the liquid sprays that they do not stain papers and fabrics in the room.

R. H. O.

High Speed Steel Tips Cemented to Low Alloy Shanks.—GEORGE SHULER. (*Iron Age*, Vol. 149, No. 16.) For many years attempts have been made to weld high speed steel to medium carbon

steel shanks in making various cutting tools. However, with the exception of the electric resistance butt welding operation as used on drills, the process was never too successful, particularly for the reason that mortality of tools was extremely high due to cracks from welding and weak bonds. Even in butt welding considerably more high speed steel is used when tips only are inserted. Furthermore, many shops are not equipped with butt welding facilities. The waste of high speed steel in making solid tools has always been great. Many tons of high speed steel have been scrapped due to breakage at the tool post and many more tons discarded due to the tool being too short for tool holders after a limited number of regrinds. These observations induced W. R. Chapin, a veteran metallurgist of Indianapolis, to see if a cementing process could not be developed which would make it possible to cement high speed steel tips on medium carbon steel shanks. This research extended through nearly a quarter of a century and resulted in many failures. Finally, Mr. Chapin perfected a metallic, powder-fine cement which gave satisfactory results under the most severe tensile tests. These tests showed that specimens broke outside the cemented joint indicating that the joint was at least as strong as the tool itself. Tool tips continued to render satisfactory service when as little as 1/16 inch remained. In many instances, the medium carbon shank seemed to act as a shock absorber, which gave a longer tool life than could be obtained from a tool made entirely of high speed steel. The new product was given the name "Cinch" steel cement and was utilized on planer knives, limestone planer tools, steel shear blades, stone and woodworking tools, and innumerable machine tool blades. Now with the coming of the war, Cinch steel cement is playing an increasingly important role in the conservation of high speed steel supplies. The toolmakers at the Timken Roller Bearing Co. plant at Canton, Ohio, report that this cement has effected the saving of at least 75 per cent. of the former high speed tool steel consumption.

R. H. O.